Banks bailout, time-consistency and distributional effects ☆

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Abstract

This paper addresses the questions of the distributional impact of banking crises and the optimal degree of public bailout, in the absence of commitment. We use an incomplete markets, heterogeneous-agents model where bankers and depositors interact and the former have a portfolio choice to make between a risky and a safe asset. Banking sector wide aggregate shocks disrupt the economy and the government can bail banks out, dampening the financial losses while benefiting both bankers and depositors. Absent a commitment device, a dynamic game is created between the government, its future selves and those who would benefit from its action. The model is calibrated to reproduce rarely occurring crises, the intensity of which would be comparable to the US 2008 subprime crisis. We compare the time-consistent equilibrium with a situation in which the government can commit to a pre-announced bailout rule. We show that the commitment case yields small welfare gains in the long run, but it would be costly to switch to it, once the transitional costs are taken into consideration.

Keywords: Banking crises, bailout, time consistency, heterogeneous-agents model.

1. Introduction

The recent financial crisis has illustrated that governments and central banks have a propensity to intervene during such large economic turmoils, especially to alleviate the stress

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on the financial and banking sectors. One typical form of intervention is the bailout operation, but as other forms of government action, this policy is subject to a time-inconsistency issue. One could imagine, for instance, that if such an intervention is anticipated, banks could coordinate their decision processes to render the bailout unavoidable. When commitment is impossible, a dynamic game is created between the benevolent planner, its future selves and those who would benefit from its action. In this paper, we use a multi-banks and depositors setting to assess the welfare and distributional implications of large banking crises and the subsequent government bailout operations when no commitment device is available. The type of crises we consider are expected to cause severe write downs on banks' balance sheets and potential losses on deposits too. Given the distributional implications, we examine the behavior of a benevolent planner in such a context, and consider the costs and gains of being unable to commit to a pre-announced policy.

We use an incomplete markets, heterogenous agents economy as our basic building block. We model the interactions between two types of agents, bankers and depositors, who differ by the type of assets they have access to. Depositors are subject to a labor market risk, are unable to borrow and have only access to a simple bank deposit, that qualifies as low-interest risk-free liquid savings. Bankers on the other hand face a similar labor market risk but have access to both a risk-free and risky investment opportunities. Moreover, bankers can use collected deposits as a leverage in addition to their own wealth when investing, as long as they abide by a prudential regulation protecting depositors. Crises disrupt the economy as banking sector wide aggregate shocks that causes losses on the risky investment, on top of the normal operational risk. A benevolent social planner, that we call the government, runs a bailout operation and levies taxes on the labor income to fund it. A key ingredient is that all commitment devices are assumed away: the government cannot bind itself to a pre-announced policy, and both the current government and the agents expect the definition of a new bailout policy once a crisis occurs. This policy is decided using a utilitarian welfare criterion on the base of a tax menu and the associated bailout ratio that is submitted to the economy.

The time-consistency issue that arises in this model can be summarized as follows: as the government cannot commit to a pre-chosen bailout rule, bankers know that the losses during a crisis will be mitigated by the fraction of their risky assets the government will agree to bail out. This pushes up the fraction of risky assets in the bankers' portfolio and create several redistributive channels: the bailout reduces the after tax wage of all agents, lowers the loss of either depositors or bankers or potentially both and a higher share of risky assets increases the total income of bankers. The redistributive impact of a bailout is thus subtle. We compare the time-consistent model to a the case where the government can bind itself to a bailout policy. We first find that when the government can commit to a tax rate and its associated bailout ratio, the optimal long-run policy welfare wise is to apply a significantly lower tax rate than in the time-consistent case. However lowering the bailout ratio in such a manner is not very efficient as most of the gains come from higher capital income. When we properly model the transition from the time-consistent equilibrium to the commitment case, and thus consider short-run effects of this policy, we find that the switch to the commitment policy is never beneficial.

Our baseline economy also displays interesting properties linked to the portfolio decision of the banks: we find that the average share of risky assets in the banks' portfolio is around 63%, which could be qualified as a moderate value given the significant risk-premium of 3.25% considered. Our simulations show that crises significantly reduce the wealth of bankers, especially the richest ones. This has an impact on the aggregate fluctuations of the model: total financial wealth sharply declines after a crisis but its recovery is quite long, precisely because bankers wish to accumulate very high levels of assets in this economy. The model also displays high concentrations of wealth in the hands of bankers and very rich depositors. Surprisingly the wealth concentration is not very much affected by the crisis and the bailout operation will have small implications regarding this matter: crises do not significantly reduce inequalities and bailouts tend to mitigate this even more.

This paper can be related to several strands of the literature. Since the 2007 financial crisis, a growing number of contributions have reconsidered central macroeconomic questions with more focus on financial intermediaries, and especially fragile banks. This gave rise to a literature where banks were more carefully modeled as opposed to simple intermediaries. Applied to monetary policy, prime examples of this literature include Gertler and Karadi (2011) and Angeloni and Faia (2013). The former carefully model the balance sheet of banks and introduce an agency problem between the financial intermediaries and the depositor to understand unconventional central bank interventions, namely absorption of (toxic) financial assets previously held by private financial intermediaries. The latter use the seminal contributions of Diamond and Rajan (2000) and Diamond and Rajan (2001) as the main building block for banks. The fragility of the banking system is considered though the increased bank run probability with the leverage the intermediary has access to and monetary policy and bank regulation are studied in this context. Our approach, although stylized, also uses an explicit modeling of banks capitalists and the management of leverage. We assume away balance sheet risk by an implicit regulation to cover deposits but instead consider a sector wide financial crisis. Moreover, the papers above do not consider the time-consistency problem that arise between the inherent risk taking of financial intermediaries and the government as the bailout operator. An adjacent literature has precisely focused on these issues. Bagehot (1873) famously provides an early discussion of the opportunity to bailout a bank in distress. Mailath and Mester (1994) investigate the incentive for a regulator to close a deposit bearing bank and show that the risk taking behavior of the financial institution is influenced by the bailout policy. They argue that the first best bailout policy is out of reach because of the lack of commitment. However this paper only consider a single bank[1]. A more recent literature has focused on the occurrence of multiple equilibria in a time-consistent framework. Acharya and Yorulmazer (2007), in a multi-bank environment, conclude that the perspective of a bailout policy leads banks to take increasingly correlated risks and illustrate the "too-many-to-fail" argument. Ennis and Keister (2009) and Farhi and Tirole (2012) also underline the fact that if a bailout is expected financial institutions react in a way that makes it optimal, but otherwise the bailout is suboptimal. Chari and Kehoe (2013) extend this literature by considering policies that could ex ante mitigate the time inconsistency issue. The papers above derive from the optimal contract literature. Our approach is much more quantitative and we consider incomplete markets, a multiplicity of banks, mobility between bankers and depositors and distributional issues. But similarly to Farhi and Tirole (2012), we study a global crisis depicted as an aggregate shock impacting the whole of the banking sector at the same time. Finally, this paper can also be related to a more methodological literature. We derive our equilibrium concept mostly from Krusell et al. (1997). Other papers have built on this approach and extended this quantitative time-consistent policy literature: Klein and Rios-Rull (2003) assess optimal fiscal policy in the absence of commitment, Krusell (2002) solves differentiable Markov equilibria in the context of redistribution policies, Klein et al.

[1]Other papers such as Rajan (1994), Mitchell (1998) or Thakor (2005) revisit the reputational argument in case of a bailout. This is not explored in the current contribution.

(2008) devise a compact characterization of the Markov-perfect equilibrium and applies it to the provision of public goods and Kankanamge and Weitzenblum (2016) characterize the time-consistent unemployment insurance policy. Even though we use a similar methodology, none of the above papers consider the bailout policy in a banking context.

The rest of the paper is organized as follows. The next section presents our reference time-consistent model. Section 3 discusses the model calibration and section 4 presents our main results. Section 5 quantifies the absence of commitment and section 6 concludes.

2. The model

2.1. Households

We assume that the economy is populated by infinitely lived households of unit mass, who can be of two types: depositors or bank capitalists. In each period, a constant exogenous fraction f of the population are bankers, in the spirit of the specification of for instance Gertler and Karadi (2011) or Angeloni and Faia (2013). A banker has a probability π^{bb} of remaining in this activity next period and thus a probability $1-\pi^{bb}$ of reverting to a depositor status. Similarly, depositors face the probability π^{ww} of remaining depositors, and the steady state distribution of agents with respect to their occupational status is such that:

$$f(1-\pi^{bb}) = (1-f)(1-\pi^{ww}) \Leftrightarrow \pi^{bb} = 1 - \frac{1-f}{f}(1-\pi^{ww})$$

Depositors inelastically supply labor and receive an exogenous wage rate w in line with their individual productivity s. Idiosyncratic productivity evolves according to an exogenous Markovian process. The labor income of depositors is taxed at rate τ. We assume that due to incomplete markets, depositors can save in a bank account, but cannot borrow. The financial wealth of a given depositor is noted a and her deposit yields an exogenous return r^b at every period. The budget constraint of a depositor, in the absence of a banking crisis, is the following:

$$a_{t+1} = (1+r^b)(a_t + s_t w(1-\tau_t) - c_t) = (1+r^b)h_t^D$$

The timing is such that wage collection and consumption expenses both take place at the beginning of the time period. a_t being the beginning-of-period financial wealth accumulated from the previous period, we will denote as depositors savings the amount $h_t^D = (a_t + s_t w(1 - \tau_t) - c_t)$ which gives rise to the risk-free return r^b.

Bank capitalists retain some features of depositors. They also inelastically supply labor, and their individual productivity is governed by the same Markovian process as the workers. Also, their labor income is taxed at the same rate τ. However, rather than saving in an outside bank, capitalists, through their own bank, have access to two different financial assets: a risk-free asset (a bill), with a maturity of a single period, and a return r^a, and a risky asset, with a stochastic return $\tilde{r}^k$ assumed to be i.i.d.. The stochastic return is idiosyncratic: its realization depends on the success of a specific investment, so that different bankers will end up with different realizations of the risky return at the same date. Their are no transaction costs, and at the beginning of the next period, the capitalist will dispose of liquid wealth, namely, money, no matter what the previous portfolio choice was. Moreover, being bankers, these agents can invest the deposits of workers too. We assume that all worker deposits are pooled and that each banker in the economy receives the same amount of deposits, noted D, such that[2]:

$$D_t = \frac{\sum_s \int h^D \Psi^D(a,s) da}{f}$$

where $\Psi^D(a,s)$ is the distribution of depositors over asset holdings and labor market statuses. Moreover, in normal times, a banker has the obligation to repay depositors their savings and the accrued interest, implying that bank runs are ruled out by assumption[3]. The budget constraint of a banker, in the absence of a banking crisis, is thus the following:

$$\begin{aligned} a_{t+1} &= (1 + (1-\eta) r^a + \eta \tilde{r}^k)(a_t + s_t w(1-\tau_t) + D_t - c_t) - (1 + r^b) D_t \\ &= (1 + (1-\eta) r^a + \eta \tilde{r}^k) h_t^B - (1 + r^b) D_t \\ &= \tilde{R}_\eta h_t^B - (1 + r^b) D_t \end{aligned}$$

with η the fraction the bank capitalist chooses to invest in the risky asset and $\tilde{R}_\eta =$

[2]Note that the decision rule h^D is written without mentioning its arguments. This is simply meant to avoid complicating the presentation. The decision rule as a function of its arguments will appear later in the text.

[3]In Appendix A, we explain the portfolio choice of the bankers and how they make sure that all worker deposits are repayed.

$(1+(1-\eta)r^a+\eta\tilde{r}^k)$ the gross return for a portfolio choice η.

To guarantee that, in the absence of a crisis, bankers would never default on the deposits, bankers face a limit on the fraction η. Assuming that the worst possible return is r^k_{min}, the following inequality has to hold:

$$(1+(1-\eta)r^a+\eta r^k_{min})h^B_t \geq (1+r^b)D_t$$

which can be re-stated as an upper bound on η, given both a_t and c_t:

$$\eta \leqslant \eta^{max} = \frac{(1+r^a)(a_t+s_t w+D-c_t)-(1+r^b)D}{(r^a-r^k_{min})(a_t+s_t w+D-c_t)} \tag{1}$$

Households derive utility out of consumption, noted c, and their preferences are additively separable over time, such that:

$$\mathcal{V} = \mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t v(c_t)\right\} \tag{2}$$

with:

$$v(c_t) = \frac{c^{1-\sigma}}{1-\sigma},$$

where β is the discount factor.

2.2. Crises

The economy is subject to crises akin to large scale financial crises. We assume that crises occur after all current consumption and saving decisions have been made. The probability of a crisis is noted χ. When a crisis occurs, a fraction φ of all the risky assets held in the banks are written down. By restricting the losses to the risky assets, we have in mind losses as, for example, those which happened in the 2008 subprime crisis. The 2009 European sovereign debt crisis, on the opposite, could be apprehended by losses on the risk-free asset held by bankers. Although substantial previous public bailouts could be one of the causes of crises on sovereign debt, modeling losses on both the risky and the safe asset would however imply a simultaneity of the crises, which would be quite counterfactual. Therefore, our modeling assumption amounts to restricting to a specific type of crisis.

We define as H^B_t the bankers's current total investment:

$$H_t^B = \sum_s \int h^B \Psi^B(a,s)da$$

where $\Psi^B(a,s)$ is the distribution of bankers over asset holdings and labor market statuses.

2.3. The government

The government runs a bailout operation in case of a crisis and levies taxes to fund it. All sources of labor income in the economy are taxed so that the total amount of taxes available at one point is:

$$T_t = \tau_t w \left(\Sigma_s s \left(\int \Psi^B(a,s)da + \int \Psi^W(a,s)da \right) \right)$$

with τ_t the tax rate. We assume that the government has a *bailout production function*, which makes it the best user of one unit of wealth in the economy for such an operation. This function defines how much can be bailed out given a certain amount of wealth collected through taxes, T_t:

$$\mathcal{B}_t = \gamma T_t^{\nu}$$

with γ and $\nu < 1$ two parameters and $\mathcal{B}$ the total amount bailed out. This technology is characterized by decreasing marginal returns: the first unit of wealth devoted to bailing out is extremely efficient, and bailing out becomes less and less efficient, as its size increases.

The government has the ability to bailout banks. By doing so, it not only offers liquidity to the banks -because the bailout is not to be reimbursed-, but it reduces the global loss by more than what it supplies, at least for bailouts of a small magnitude. In this highly stylized setup, it is relatively easy to tell stories which would be consistent with our model: by bailing out a given bank, it increases the value of the bank, and it also prevents other financial institutions, lenders to this bank, from writing down a higher fraction of their assets. Here, interbank links are not explicitly modeled, but the bailout technology is meant to capture the efficiency of channelling financial resources to banks in times of crisis. The bailout ratio is measured as:

$$\theta = \frac{\mathcal{B}_t}{\varphi \Sigma_s \int \Sigma_k \pi_r^k (1 + r^k) \eta h^B \Psi^B(a,s)da}$$

where $\sum_s \int \sum_k \pi_r^k (1+r^k)\eta h^B \Psi^B(a,s)da$ measures the amount of aggregate risky assets held in the economy before a crisis but after consumption and saving operations have been performed by agents for the current period and π_r^k is the probability that the return on the risky asset is k.

Given the level of the public bailout, we assume that the bailout ratio is identical for all banks -although banks are of various sizes, depending on the current wealth of their owners. Therefore, the net assets held by a given bank, after the occurrence of the crisis and the bailout, at the beginning of the time-period, are worth:

$$\begin{aligned}(a_t + s_t w(1-\tau_t) + D_t - c_t)\left[\eta\left(1-\varphi\left(1-\theta\right)\right)\left(1+\tilde{r}\right) + (1-\eta)(1+r^a)\right] \\ = \left(h^B(a,s)\right)\left[\eta\left(1-\varphi\left(1-\theta\right)\right)\left(1+\tilde{r}\right) + (1-\eta)(1+r^a)\right]\end{aligned}$$

Bankers are under the obligation of paying back depositors what they owe them, up to the assets still held after the crisis, and they will only be entitled to keep what remains. It could well be that the net asset value of the bank is not enough to cover all deposits, as the prudential rule only guarantees savings in normal times. In such a case, bankers loose their entire financial wealth. The loss of depositors then depends on the specific value of the net assets held by a given bank. To avoid creating heterogeneity among depositors, we assume that whenever a crisis occurs, an *ex post* insurance, collected among all remaining deposits, is redistributed, so that the net loss ratio is equalized among all depositors. Consequently, a depositor's net financial wealth is independent from the specific bank which hosts her account.

The government chooses the tax rate (and consequently the bailout ratio) in order to maximize the utilitarian criterion, as follows:

$$\Phi(\Psi) = \arg\max_{\tilde{\tau}} \sum_s \sum_{i=D,B} \int_{a_{min}}^{a_{max}} V^i\left(a, s, \Psi, \tilde{\tau}\right) T_{\tilde{\tau}}\left(\Psi^i(a,s)\right) da$$

where $V^i\left(a, s, \Psi^D, \Psi^B, \tilde{\tau}\right)$ and $T_{\tilde{\tau}}$ are respectively the expected intertemporal utility[4] of agent of type i and the distribution transformation representing the impact of the loss φ, and the

[4]The presence of Ψ^D, Ψ^B among the arguments of the value function will be made clear in the next subsection.

subsequent bail-out associated with the tax rate $\tilde{\tau}$. The government cannot commit to a specific level of future bailout. Whenever a crisis occurs, the bailout is re-computed. With the recursive formulation of the agents program, we will see that each time the government decides on the current level of bailout, it takes as given its future decisions. This corresponds to the behavior of a government unable to commit to future choices.

2.4. Recursive formulation and equilibrium

The state of the economy is exhaustively described by the distributions of agents $(\Psi^B(a,s), \Psi^W(a,s))$ and the current tax rate τ. The recursive formulation of the workers program is:

$$
\begin{aligned}
&V^D(a,s;\Psi^B(a,s),\Psi^D(a,s),\tau) = \qquad (3)\\
&\quad \max_{c,a'}\Bigg\{u(c)+\beta\Big[\\
&\quad \chi\Big(\pi^{dd}V^D(a'_\chi,s';\Psi'_C{}^B(a,s),\Psi'_C{}^D(a,s),\tau'\left(\Psi^B(a,s)\right),\Psi^D(a,s),\tau))\\
&\quad +(1-\pi^{dd})V^B(a'_\chi,s';\Psi'_C{}^B(a,s),\Psi'_C{}^D(a,s),\tau'\left(\Psi^B(a,s)\right),\Psi^D(a,s),\tau))\Big)\\
&\quad +(1-\chi)\Big(\pi^{dd}V^D(a',s';\Psi'^B(a,s),\Psi'^D(a,s),0)\\
&\quad +(1-\pi^{dd})V^B(a',s';\Psi'^B(a,s),\Psi'^D(a,s),0)\Big)\Big]\Bigg\}\\
&\quad s.t.\ a'=(1+r^b)(a+sw(1-\tau)-c)\\
&\quad a'_\chi=(1+r^b)(a+sw(1-\tau)-c)(1-\kappa\left(\Psi^B(a,s),\Psi^D(a,s),\tau\right))\\
&\quad a'\geq 0
\end{aligned}
$$

$\Psi'^B(a,s), \Psi'^D(a,s)$ (resp. $\Psi'_C{}^B(a,s), \Psi'_C{}^D(a,s)$) are the next period distributions of agents in the absence (resp. in the case) of a crisis. They are obtained from the current distributions of agents, to which the saving rules (the values of a' computed from the above program, for all possible states of the economy) are applied.

$\tau'(\Psi^B(a,s)), \Psi^D(a,s), \tau)$ associates, to the current distributions, the public bail-out in terms of tax rate, in case a crisis occurs at the outset of the next period. In case of a crisis, the loss φ bears on the value of the risky assets held by the bankers. The level of the bailout tax[5] τ is then computed to maximize the utilitarian criterion applied to these distributions.

$\kappa\left(\Psi^B(a,s),\Psi^D(a,s),\tau\right)$ is the function associating to each possible current state of the economy, the average loss on deposits in case a crisis occurs at the beginning of the next period. As mentioned above, deposits are guaranteed up to the net asset held by bankers. This fraction is thus obtained as follows:

$$
\begin{aligned}
&\kappa\left(\Psi^B(a,s),\Psi^D(a,s),\tau\right) = \\
&\quad \frac{1}{f(1+r^b)D}\int\sum_s\sum_k \pi_r^k \Psi^B(a,s) max\left[(1+r^b)D - \left\{(1-\varphi(1-\theta))\eta(1+r^k)\right.\right. \\
&\qquad\qquad\qquad\qquad \left.\left. +(1-\eta)(1+r^a)\right\}(a+sw(1-\tau)+D-c^B(a,s));0\right] da
\end{aligned}
$$

where $\theta = \theta(\Psi^B(a,s)),\Psi^D(a,s),\tau)$ associates, to the current state of the economy, the fraction of the loss incurred during the crisis, which will be bailed-out. This fraction θ will apply to the post-crisis distribution of agents at the beginning of the next period, but, as of date t, the information set is $\left\{\Psi^B(a,s)),\Psi^D(a,s),\tau\right\}$. The θ function, like the τ' one, implicitly comprises the law of motion of the distribution of agents between the two consecutive dates. In other words, agents associate to the current distribution, the future one in the absence of a crisis, to which they apply the losses due to the crisis, and the public bail-out.

The recursive formulation of the bankers' program is:

[5] The tax rate τ among the arguments of the function is the one applying during the current period, while the function τ' computes next period's tax rate, in case of a crisis.

$$
\begin{aligned}
&V^B(a,s;\Psi^B(a,s),\Psi^D(a,s),\tau) = \qquad (4)\\
&\qquad \max_{c,a',\eta}\Bigg\{u(c)+\beta\Bigg[\\
&\qquad \chi\Big(\pi^{bb}\sum_k \pi_k^r V^D(a_{\chi,k},s';{\Psi'_C}^B(a,s),{\Psi'_C}^D(a,s),\tau'\left(\Psi^B(a,s)\right),\Psi^D(a,s),\tau))\\
&\qquad +(1-\pi^{bb})\sum_k \pi_k^r V^B(a'_{\chi,k},s';{\Psi'_C}^B(a,s),{\Psi'_C}^D(a,s),\tau'\left(\Psi^B(a,s)\right),\Psi^D(a,s),\tau))\Big)\\
&\qquad +(1-\chi)\Big(\pi^{bb}\sum_k \pi_k^r V^D(a'_k,s';\Psi'^B(a,s),\Psi'^D(a,s),0)\\
&\qquad +(1-\pi^{bb})\sum_k \pi_k^r V^B(a'_k,s';\Psi'^B(a,s),\Psi'^D(a,s),0)\Big)\Bigg]\Bigg\}\\
&s.t.\ a'_k = (1+\eta r^k+(1-\eta)r^a)(a+sw(1-\tau)+D-c)-(1+r^b)D\\
&\qquad a'_{\chi,k} = max\left((a+sw(1-\tau)+D-c)\left[(1-\eta)(1+r^a)\right.\right.\\
&\qquad \left.\left.+\eta(1-\varphi(1-\theta(\Psi^B(a,s),\Psi^D(a,s),\tau)))\,(1+r^k)\right]-(1+r^b)D;0\right)\\
&\qquad a'\geq 0\\
&\qquad \eta\leqslant min\,(1,\eta^{max})\\
&\qquad D=\frac{\sum_s\int h^D(a,s)\Psi^D(a,s)da}{f}
\end{aligned}
$$

η^{max} is the highest value for the risky investment share compatible with the regulatory constraint that, in the absence of a crisis, bankers should always pay back the depositors the full amount D and the accrued interest r^b. In particular, η^{max} depends on the value of the lowest possible realization for the risky return, r^k_{min}.

The state of the economy being a set of distributions, we need, in our numerical computations, to approximate it with an object of much smaller dimension. In the spirit of Krusell and Smith (1998), we choose to consider the average financial wealth held in the economy, A_t, as a sufficient statistics. A_t, summed over all agents, is the beginning-of-period financial wealth carried over from the previous period:

$$
A_t=\sum_{i=D,B}\sum_{s\in S}\int_A a\Psi^i(a,s)da
$$

The core of the recursive problem then consists of:

1. the laws of motion in the absence of a crisis at the outset of the next period: $A_{t+1} = \Gamma^0(A_t, \tau_t)$, and in the presence of a crisis at the outset of the next period, net of the bailout: $A_{t+1} = \Gamma^C(A_t, \tau_t)$;
2. the laws $D_t = \Delta(A_t, \tau_t), \kappa_t = \kappa(A_t, \tau_t)$. These approximations are meant to associate to the state of the economy $((A_t, \tau_t))$, the level of deposits[6], the average share of risky assets and the average loss for depositors in case of a crisis.

The depositors and the bankers programs can be rewritten as follows:

$$
\begin{aligned}
V^D(a, s, A, \tau) = & \\
& \max_{c,a'} \Bigg\{ u(c) + \beta \Bigg[\\
& \quad \chi\Big(\pi^{dd} V^D(a'_\chi, s', A'_C, \tau'(A, \tau)) + (1-\pi^{dd}) V^B(a'_\chi, s', A'_C, \tau'(A, \tau))\Big) \\
& \quad +(1-\chi)\Big(\pi^{dd} V^D(a', s', A', 0) + (1-\pi^{dd}) V^B(a', s', A', 0)\Big)\Bigg]\Bigg\} \\
& s.t.\ a' = (1+r^b)(a + sw(1-\tau) - c) \\
& \quad a'_\chi = (1+r^b)(a + sw(1-\tau) - c)(1-\kappa(A, \tau)) \\
& \quad a' \geq 0
\end{aligned}
\tag{5}
$$

[6]Deposits are not a state variable, but the beginning-of-period wealth of depositors, similarly to total financial wealth A_t, is a predetermined variable which, in principle, should be included as another state variable. To restrict the dimension of the problem, since the model is numerically computed, we simply omit to regard wealth of depositors as a state variable, and consider that its evolution can be predicted with enough accuracy through the knowledge of A_t.

$$V^B(a,s,A,\tau) = \tag{6}$$

$$\max_{c,a',\eta}\Bigg\{u(c)+\beta\Bigg[$$

$$\chi\Big(\pi^{bb}\sum_k \pi^r_k V^B(a_{\chi,k},s',A'_C,\tau'(A,\tau))+(1-\pi^{bb})\sum_k \pi^r_k V^D(a'_{\chi,k},s',A'_C,\tau'(A,\tau))\Big)$$

$$+(1-\chi)\Big(\pi^{bb}\sum_k \pi^r_k V^B(a'_k,s',A',0)+(1-\pi^{bb})\sum_k \pi^r_k V^D(a'_k,s',A',0)\Big)\Bigg]\Bigg\}$$

$$s.t.\ a'_k = (1+\eta r^k+(1-\eta)r^a)(a+sw(1-\tau)+D-c)-(1+r^b)D$$

$$a'_{\chi,k} = max\left((a+sw(1-\tau)+D-c)\left[(1-\eta)(1+r^a)\right.\right.$$

$$\left.\left.+\eta(1-\varphi(1-\theta(A,\tau)))(1+r^k)\right]-(1+r^b)D;0\right)$$

$$a' \geq 0$$

$$\eta \leqslant min\,(1,\eta^{max})$$

For numerical tractability, all these rules are assumed to be linear:

$$\Gamma^0(A_t,\tau_t) = \alpha_0^{\Gamma 0}+\alpha_1^{\Gamma 0}A_t+\alpha_2^{\Gamma 0}\tau_t$$
$$\Gamma^C(A_t,\tau_t) = \alpha_0^{\Gamma C}+\alpha_1^{\Gamma C}A_t+\alpha_2^{\Gamma C}\tau_t$$
$$\Delta(A_t,\tau_t) = \alpha_0^{\Delta}+\alpha_1^{\Delta}A_t+\alpha_2^{\Delta}\tau_t$$
$$\kappa((A_t,\tau_t) = \alpha_0^{\kappa}+\alpha_1^{\kappa}A_t+\alpha_2^{\kappa}\tau_t$$

2.5. The time-consistent equilibrium

The resolution of the above programs provides the individual decision rules $c^D(a,s,A,\tau)$, $c^B(a,s,A,\tau)$ and $\eta(a,s,A,\tau)$. From these rules, one can build the investment rules:

$$h^D(a,s,A,\tau) = a+sw(1-\tau)-c^D(a,s,A,\tau)$$
$$h^B(a,s,A,\tau) = a+sw(1-\tau)-c^B(a,s,A,\tau)+\Delta(A,\tau)$$

The equilibrium of the economy consists of the value functions $\left\{V^D(a,s,A,\tau),V^B(a,s,A,\tau)\right\}$, the decision rules $\left\{h^D(a,s,A,\tau),h^B(a,s,A,\tau),\eta(a,s,A,\tau)\right\}$, and the aggregate laws $\left\{\Gamma^0(A_t,\tau_t),\Gamma^C(A_t,\tau_t),\Delta(A_t,\tau_t),\kappa((A_t,\tau_t)\right\}$ such that:

1. Given the laws $\left\{\Gamma^0(A_t,\tau_t),\Gamma^C(A_t,\tau_t),\Delta(A_t,\tau_t),\kappa((A_t,\tau_t)\right\}$, $\left\{V^D(a,s,A,\tau),V^B(a,s,A,\tau)\right\}$ are the value functions obtained from programs (5) and (6) and $\left\{h^D(a,s,A,\tau),h^B(a,s,A,\tau),\eta(a,s,A,\tau)\right\}$ are the associated individual decision rules;
2. Given the individual decision rules, the path of the economy simulated over a large number of periods is consistent with the ex ante postulated aggregate laws;
3. whenever a crisis occurs, the government maximizes the utilitarian criterion: and this choice is consistent with the above laws.

3. Calibration

We now describe how the values for the various parameters governing the labor market transitions, the process for the risky return or the agents' preferences have been set in the benchmark case. The model period is the year. The benchmark calibration is summarized in Table 1.

Labor productivity follows a first order autoregressive process with an auto-correlation coefficient ρ_s of 0.3 and a standard deviation of innovation $\sigma_s = 0.2$. These values fall on the lower side of similar calibrations in macroeconomic models[7], because these models target the reproduction of intergenerational transmission of qualifications and productivity –or, equivalently, they incorporate, in a very stylized way, all the sources of variations in labor income between agents–, while our goal is quite different. Indeed, the mobility between depositors and workers will endogenously produce a new source of heterogeneity, which will substantially contribute in shaping wealth inequalities. This is why we have chosen to preserve the original calibration of the labor income process introduced by Aiyagari (1994). We then use the Tauchen procedure (Tauchen (1986)) to approximate the AR(1) process with a three-states Markov chain. We obtain the following three states $\{0.54881, 1.0, 1.82212\}$ and transition matrix:

$$\Pi^{ss} = \begin{pmatrix} 0.26468 & 0.72938 & 0.00594 \\ 0.05792 & 0.88415 & 0.05792 \\ 0.00594 & 0.72938 & 0.26468 \end{pmatrix}$$

[7] See for instance Domeij and Heathcote (2004).

Parameter	Value	Description
σ	3.0	Relative risk aversion
w	1	Wage
ρ_s	0.3	Labor productivity persistence
σ_s	0.2	Labor productivity innovation std.
ν	0.5	Bailout elasticity
γ	0.8	Bailout efficiency
χ	0.0111	Crisis occurrence probability
φ	23%	Crisis loss rate
r^b	0.5%	Deposit rate
r^a	0.5%	Bank safe rate
$\tilde{r}^k$	{-15%,1.5%,20%}	Bank risky rates
π^{r^k}	{0.2,0.5,0.3}	Bank risky rates probabilities
π^{bb}	0.95	Probability of remaining a banker
β	0.96895	Discount factor
f	10%	Fraction of banks

Table 1: Benchmark calibration values

The risk-free interest rate is set to $r^a = 0.5\%$ and depositors receive the same rate $r^b = 0.5\%$ on their bank deposits. Long time-series suggest that the risk-free rate is around 2% (see, for instance, Haliassos and Michaelides (2002)), but deposits pay a much lower real rate, especially in the last 15 years, and we wished to equalize both. The process governing the risky return is calibrated so as to give rise to an equity premium worth 3.25% and a standard deviation of 0.12 (somewhat smaller than the usually retained value of 0.15 (see Cocco et al. (2005) for instance). The risky investment yields a low, medium and high return, with values $\tilde{r}^k = \{-15\%, 1.5\%, 20\%\}$ and the associated probabilities: $\pi^{r^k} = \{0.2, 0.5, 0.3\}$. The low return is considerably negative and yet, as we will see, this will not excessively deter bankers from taking risks.

The proportion of bankers, as well as the transition probabilities between banker and depositor statuses, should also play a crucial role in shaping the exogenous sources of heterogeneity. Bankers have access to assets yielding potentially high returns, and they can use other people's deposits as a leverage. A strict way of apprehending this would lead us to include, in the definition of the banker type, all agents holding stocks of commercial banks[8]. We could have a broader view of what is called a banker by including all agents having access to leverage, even though it may not consist of real cash deposits. All in all, there is a broad range for the fraction of bankers, f, which we would consider as admissible. Simulations show how strongly the concentration of wealth (as measured, for instance, by the proportion of the global financial wealth owned by the top 1% of agents) depends on f. We set $f = 10\%$ and an average duration of remaining banker of 20 years. This choice gives a fairly good fit of the US wealth distribution with a slight overshoot. A better fit would have been possible with an increased fraction of bankers. However, even with our broad view on the banker status it seemed unrealistic to have a much larger fraction of bankers. In the next section, we will present detailed distributional characteristics of the baseline equilibrium.

As for the crisis characteristics, we need to calibrate the probability of a crisis χ, φ, the

[8]A stockholder does not participate actively in the investment policy but rather, delegates her authority to the manager. Different stockholders would not have identical individual preferences in terms of investment policies, so that the question of the aggregation of their preferences and the delegation of authority to the manager, should in principle be addressed. This is clearly not our stance, so our modelling is not fully consistent with bank stocks being freely exchangeable on the market.

gross loss on risky assets, and the public bailout function parameters γ and ν. We impose $\chi = \frac{1}{90}$, because we have in mind large banking crisis occurring very rarely. We choose $\nu = 0.5$ to guarantee that the bailout efficiency is initially very high and that the bailout, endogenously chosen by the government, will not be too high[9]. We then wish to match two relevant statistics extracted from the literature on the 2008 US subprime crisis: the average net loss of bankers, as measured by the total amount of write-downs undergone, and the fraction of the gross loss avoided by the bailout. Data on the 2008 US banking crisis are numerous, and the complexity of the true balance sheets of US banks leaves room for interpretation when it comes to translating it in terms of the model structure. The fraction of total assets held by US banks written down has been evaluated between (around) 5% (He et al. (2010)) and 15% (IMF (2009)). We will simply take the average value of these two estimates and consider that, by then (around April 2009), all the bailout had been implemented, although we realize this might be a strong assumption. Therefore, in our benchmark simulation, we target a ratio of net loss relative to all the assets held by banks to be equal to 10%. The Paulson plan, as we know, was initially set at 700 billion USD, but only a part of it was used (431 billions, according to CBO, 2012). It is also important to recall that in our model, the bailout is a pure redistribution to banks, not a loan to be reimbursed. Consequently, it should be apprehended as a net cost to the government. This seems very difficult to quantify, given that the public authorities decided to relieve banks of some of their toxic assets. There is no reason to consider that these asset values all went down to zero. Quantitative evaluation range from 34 to 68 billion USD (again, CBO, 2012).

The efficiency of the bailout is such that the net value of the assets held by bankers has increased substantially more than the net cost borne by the taxpayer, but we are at a loss as to how precisely quantify this effect. Let us consider that this amount has made possible the avoidance of a 5% loss due to write downs (the average efficiency of the bailout being here 5). This implies that the pre-bailout write down ratio was 15%, and the bailout has relieved bankers of a third of their incurred losses. Given the evaluation of the value of the

[9]With an elasticity lower than 1, for large amounts of bailout, the marginal return would be smaller than one. As the bailout benefits bankers more than depositors, especially when it is large, the government will not choose to tax uniformly all agents –among which, many are poor with high marginal utility of consumption– to inefficiently redistribute it to rich agents.

assets held by U.S. commercial banks-around 11'000 billions USD according to He et al. (2010)-, the bailout operation has avoided a 500 billion global loss. As compared to the net cost (evaluated in the $[32; 68]$ billion USD interval, it would imply a huge efficiency of the operation. This is why we have retained the following values: 100 billion USD for the net cost, and 500 billion USD for the amount bailed out.

The bailout cost, representing around 1% of the GDP, should consequently materialize as a tax on labor income of around 2%. We will not target this tax rate, because we use too few exogenous parameters, but we will simply, a posteriori, check that the obtained tax rate -endogenous in our model- will be in the correct range. In the end, we simply adjust φ and γ to match the net losses over all assets (10%) and a bailout ratio of one third. We obtain $\varphi = 23\%$ and $\gamma = 0.8$. The associated tax rate generated by our baseline model is 3.6%, substantially higher than the above calculation, but the orders of magnitude are the same. Finally, we need to make sure that the global wealth held by agents (both depositors and bankers) is in line with the data. We restrict ourselves to financial wealth (putting aside real estate) and for the SCF[10], we found that the annual ratio of financial wealth over labor and replacement income was 3.10. We adjust the discount factor β in order to match this value and consequently set $\beta = 0.96895$.

4. The baseline equilibrium

We here present the simulation results of the calibrated model. Table 2 summarizes our calibration results in terms of wealth statistics compared to US data as reported by Kuhn and Rios-Rull (2013). The model is able to reproduce the fat right tail of the wealth distribution with ease and is close to the statistics for the US distribution of wealth. The Gini index is also very close to its data counterpart.

Table 3 details the main aggregate results. We see that the bailout costs 0.036 and produces 0.15. The overall efficiency of the operation is $\frac{\mathcal{B}}{T} \simeq 4$, and its marginal efficiency is $\frac{d\mathcal{B}}{dT} = \gamma \nu T_t^{\nu-1} \simeq 2$. We see that bankers are on average around 4 times richer than depositors. Although bankers have access to a much higher return, and therefore are considerably more incited to save large amounts, the mobility between the two statuses guarantees that there will not be too high a discrepancy between the two average wealth levels. The average share

[10] Using data from 2013 SCF Chartbook and Bricker et al. (2014).

	W. Gini	Percentage wealth held by top 1%	5%	10%
Model	0.81	40.0	67.5	77.2
U.S. Data	0.85	35.5	62.9	75.0

Table 2: U.S. and model wealth statistics

of risky assets, η^{av} is at 63.0%. It is not too high, despite the substantial risk premium. η^{av} includes the leverage due to the deposits: this fraction is computed relatively to the total investment of the banker, including deposits, and not only the banker's own wealth. The prudential constraint ensuring that, in the absence of a crisis, deposits will always be paid back, also plays a role in shaping this result. Consider a banker with little wealth (for instance, a depositor who was unlucky enough to draw unfavorable productivity shocks in the recent past, and who has just become a banker). The average deposit she will collect can be high, as compared to her own wealth, so that the banker cannot choose too risky portfolios. We have computed the share of bankers for whom the actual choice on η is hitting the prudential constraint. These cases are informative of agents who are forbidden to choose higher values of η from a regulatory point of view. Bankers, by themselves, could also choose moderate values of η which, we will see and in line with the usual intuition, happen for very rich bankers. The average loss on deposits is 0.4%: its low value directly follows from (i) the rule that, in case of a crisis, deposits are served first and (ii) the fact that the loss only affects risky assets. Indeed, when a banker is wealthy, the capital she brings to her bank can be considerably higher than the collected deposits. In this case, even if risky assets suffer losses (due both to the idiosyncratic shock on the risky return and the loss following a crisis), all deposits will be guaranteed. Loss on deposits are most likely to occur for bankers with little wealth. In this case, the prudential rule will most certainly apply, restraining their propensity to buy risky assets. With a low fraction of risky assets, losses will in turn be moderate, so that deposits should not excessively suffer from the crisis. On the opposite, the average loss on bankers' wealth is 28.3%. This is not surprising, given that the loss computed as a fraction of the total assets has been calibrated to 10%, and given that depositors are barely concerned with crises losses.

Figure 1 plots the individual portfolio choice $\eta(a, s, A, \tau)$ for the three different produc-

Tax rate (in %)	3.6
Avg. wealth	3.17
Avg. wealth (depositors)	2.16
Avg. wealth (banker)	1.02
Per Capita wealth, depositors	2.39
Per Capita wealth, bankers	10.15
Avg. Deposits	2.10
Avg. bailout ratio (in %)	33.9
Avg. bailout level	0.15
Avg. loss on deposits (in %)	0.4
Avg. loss bankers (in %)	28.3
Avg. Eta (in %)	63.0

Table 3: Aggregate statitics of the baseline model

tivity levels, for $\tau = 0$ (no crisis occurred at the beginning of the current period) and for a value of $A = 3.14$ (very close to its mean, 3.17). We see that, for low levels of wealth, bankers will opt for low shares of risky assets. Bankers with no wealth and the lowest level of productivity do not save at all: they are forced to buy exclusively safe assets, to guarantee the hosted deposits. When the productivity is higher, agents save, as is quite common in the precautionary savings literature (Carroll (1996) for instance). They choose to save at least 5% (resp. 26%) with the intermediate (resp. the high) productivity. All three curves are increasing in financial wealth, until the individual wealth reaches values well above 10. From the distributions of agents, we know that a proportion 83% of bankers are located in the increasing par of the η curves. For higher wealth levels, lower than $a = 235$, the no short-selling constraint is binding. Finally, for very high wealth levels (this concerns a fraction 0.3% of bankers), the share of risky assets is decreasing with the wealth.

This behavior is very common in the portfolio choice literature (see Campbell and Viceira (2002) for instance). The total intertemporal wealth of the agent consists of both her current stock of financial wealth, and the expected discounted future flow of labor earnings. As the current stock of financial wealth increases, the proportion which the future labor earnings represents sharply decreases. As the labor market earnings risk is independent from the risk on the assets return, agents with little current wealth are inclined to chose high ηs, because their current wealth is small, as compared to the future earnings. Even if they buy only risky assets, the risk ought to be calculated with respect to the global intertemporal wealth. Conversely, as bankers become very rich, their financial wealth represents a high proportion of the total intertemporal wealth. They then chose to reduce the proportion of risky assets.

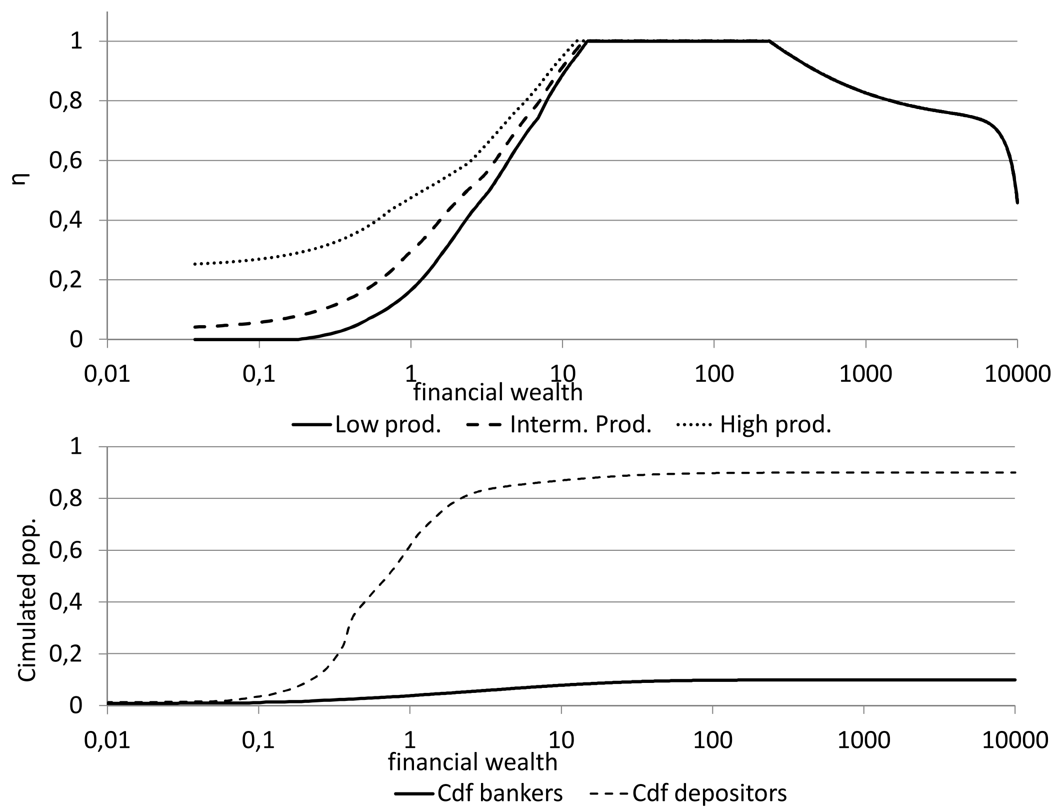


Figure 1: Individual decisions on η

Figure 1 plots the average cumulative density function[11] of bankers and depositors, with respect to their individual financial wealth. The distribution features a fat right-tail (not really visible on a cdf), which would appear more distinctly on a density function. It is due to the

[11]Because of the aggregate (crisis) shock, the economy is fluctuating, without ever reaching a steady state. What we call an average distribution simply consists in simulating the path of the economy over a long time span -90'000 years- recording the obtained distributions at each time period, and computing the average distriution.

risk premium. The expected risky return $E(1+\tilde{r}) = 3.75\%$ is such that $\beta E(1+\tilde{r}) > 1$: the incentive to save, in this infinitely-lived agents setup, remains very strong for bankers. What has guaranteed the numerical convergence[12] of the distribution, is the banker-depositor mobility: rich bankers have accumulated large amounts of wealth, because they have been very lucky in the past, and have had the chance to remain bankers for a long time. The probability of remaining banker however decreases, at a geometrical rate, as time goes by, which limits their wealth accumulation.

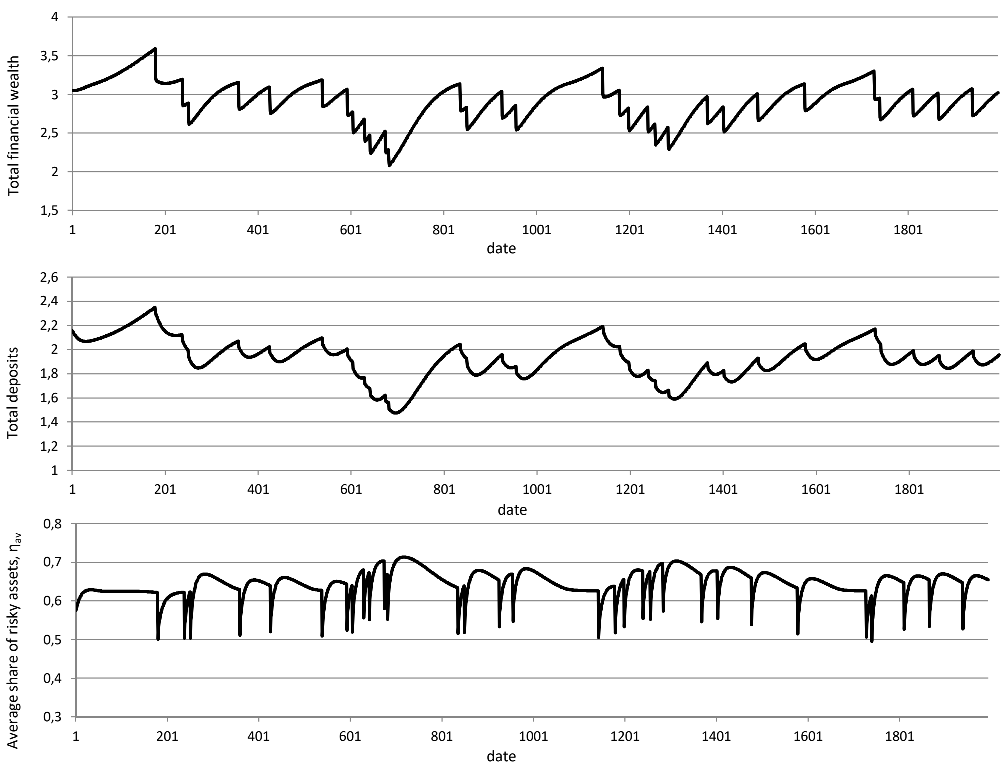


Figure 2: Dynamics of aggregate variables

As reported in figure 2, we can visualize the aggregate fluctuations by plotting the path of

[12]Note that we have not proved that the ergodic distribution of agents is bounded, nor have we proved that, even if it were not, a bounded distribution could approximate the true distribution, up to an arbitrary degree of accuracy.

some variables of interest, over a certain time-span. As crises occur in the baseline case every 90 years on average, those fluctuations are represented over 2000 years, in order to include enough different crises. As can be seen, sometimes, two consecutive crises are separated by only a few years, and sometimes, hundreds of years pass before the next crisis bursts out. This has considerable macroeconomic implications, regarding the wealth accumulation. After a crisis, the recovery in terms of global savings is initially strong, but it takes an extremely long time, before the total financial asset really stops increasing. This is due to the process governing the risky asset return: as bankers are incited to save up to very high levels of wealth, the distribution needs a long time before it stabilizes. When a crisis occurs, losses for bankers are substantial, in particular for rich ones. The process of saving to make up for the loss then starts again. Quantitatively, this implies that the total financial wealth is characterized by considerable dispersion ($\sigma_A = 0.41$).

The dynamics of deposits is similar, in that crises also cause deposits to fall, but (i) by a much smaller proportion at the outset of the crisis and (ii) deposits continue to decrease a few periods afterwards. From the above statistics, we know that the average loss for depositors is rather small, so it is no wonder that deposits should initially decrease less than total wealth. Later on, because of the depositor-banker mobility, bankers flowing into the depositor type are substantially less wealthy than prior to the crisis, while the average wealth of depositors flowing out of this type has been little affected. This justifies why deposits decrease for a few model periods. As was the case for bankers, when no crisis occurs for a long time, deposits keep increasing: this evolution is not so much driven by the savings of depositors -the risk-free rate on their deposits is small, so that they do not save very much- than by the savings of the bankers, together with the mobility between the two statuses: if bankers are getting richer, so will depositors.

The dynamics of the average share of risky assets[13] is interesting, because of its non-monotonicity. As a crisis occurs, bankers are severely hit, and a significant proportion of them (83%) are located on the increasing part of the decision rule on the share of risky

[13] computed, not as the mean of the individual shares, but as the total investment in risky assets over the sum of the total investments in risky and risk-free assets

assets. This implies that, after the crisis, they will not only be poorer, but they will also invest a lower fraction of the assets held by the banks in risky ones. After this fall, the recovery in terms of the portfolio choice is quite fast. Later on, the fraction of risky assets begins to revert back to lower levels. This non-monotonic behavior proceeds from the non-monotonic shape of the η decision rule: as bankers get richer during a recovery, more and more will reach the part where the decision rule in decreasing. These two effects operate in opposite directions. What the dynamics of η reveals, is that as the crisis bursts out, the impact of the portfolio adjustment of poor bankers dominates, while after some time, these bankers have rather quickly recovered, and the long-lasting savings of the richest bankers and their impact on the average η dominate.

Table 4 presents the coefficients of the linear relations approximating the laws of motion of total financial wealth, and the rule associating to (A, τ), the level of deposits and he fraction of deposits lost.

	α_0	α_A	α_τ
Law (crisis)	0.156	0.854	1.237
Law (no crisis)	0.015	0.997	-1.1
D_t	0.079	0.638	3.821
κ_t	0.003	0.000	-0.038

Table 4: Coefficients of the laws of motion

Regarding the laws of motion of total wealth, the loss during a crisis clearly appears on the lagged coefficient on A. Conversely, in the absence of a crisis, this coefficient is very close to one. This shows that the accumulation process in normal times should take considerable time, and lead the economy to rather high levels of wealth. Deposits are strongly linked with total wealth, which is the quantitative confirmation of what has been observed in the simulated time-series. The positive coefficient on the current tax rate materializes the positive impact of the size of bailout on the level of deposits.

4.1. The distributional impact of banking crises

We here wish to shed light on the distributional effect of crises. The model contains several channels through which crises will alter wealth inequalities. First, losses, net of the bailout, are a lot higher for bankers than for depositors. However, not all bankers are wealthy, nor are all depositors poor. Clearly, very wealthy bankers will suffer a lot from the crisis[14], while poor depositors will not. Rich depositors -who were bankers in the recent past- will undergo a very low loss ratio; in terms of relative wealth, they will be the winners in case of a crisis. Conversely, poor bankers will be those for whom crises are the most costly, at least from a perspective ranking agents relatively in terms of wealth.

To quantify these various effects, we present in Table 5 some wealth quintiles just before the outset of a crisis, and just after it, for a particular date, such that the aggregate wealth was very close to its mean. This is an illustration, among a large variety of possible distributions prior to the crisis.

		Percentage wealth held by top			
	1%	5%	10%	20%	50%
Before the crisis	40.2	68.1	77.6	84.7	94.6
After the crisis	41.5	68.6	77.2	84.2	94.5

Table 5: Distributional impact of a crisis

What Table 5 reveals, is that the concentration of wealth is not very much affected by the occurrence of the crisis. More surprisingly, the concentration of wealth, as measured by the proportion held by the top 1%, is in fact increased by the crisis. This apparent paradox is best explained by looking at the composition of the top 1% of richest agents. Before the crisis, 58% of this quintile are depositors, and this proportion climbs to 67% after the crisis. This means both that among the richest agents, depositors account for a majority -who will hardly suffer any loss- and that, posterior to the crisis, they will be even more present.

This result rests on the assumption regarding the mobility between depositors and bankers. Yet, each status is quite persistent in time (the expected duration of remaining banker is 20 years, and its equivalent for depositors is 180 years), so it was not obvious from the beginning.

What could be the implications of the distributional impact of crises, and of the size of the bailout, on welfare ? First, if crises do not considerably affect wealth concentration,

[14] Although the individual share of risky asset is decreasing in wealth for rich bankers, so that the loss, measured in proportion with the total wealth, will also be decreasing in wealth.

then the impact of the bailout -which dampens the losses- should be even smaller. If one had the intuition that crises hit relatively more the richest agents, and would reduce wealth inequalities, then bailout measures, on the contrary, would tend to increase inequalities. What this analysis suggests, is that the effect would at best be very small. This does not mean that bailout operations are not anti-redistributive in terms of welfare: we can only say that the relative inequality is rather stable, not that the poorest agents are not harmed most by the policy, as we have left aside the question of the tax collection here.

5. Quantifying the absence of commitment on welfare

The model structure has been designed so that each government bailout decision, undertaken at the outset of the crisis, aims at maximizing the current social welfare, given expectations regarding the future evolution of the economy. This implies that the bailout choice can be different from one which would be decided upon earlier in time, and to which the public authorities would commit. This latter arrangement, which will be denoted as the commitment case, is better known as the Ramsey solution. In this economy, the Ramsey solution would entail a bailout-rule decided once and for all over the whole expected future time-path of the economy, conditional on its initial state. Such a rule is very difficult to characterize numerically, so instead, we will consider simpler commitment rules, where the tax rate in case of a crisis is constant (and therefore independent from the current state of the economy).

The costs of being unable to commit to a pre-chosen rule can be considerable in this setup. The bankers being rational, they know that, if a crisis should be set off at the beginning of the next date, the government will bailout a certain fraction of the losses bearing on the risky assets held by the banks. This reduces the fear of the bankers regarding the occurrence of the crisis, therefore pushing up their portfolio choice toward risky assets. This gives rise to several redistributive channels. First, a larger amount of bailout will uniformly reduce the after-tax wage of all agents. Secondly, considering a given bank, the bailout will lower the losses of either the depositors, or of the bankers, or possibly of both in certain proportions. For instance, if the banker is very wealthy, deposits represent a small amount of the total liabilities of her bank, so that the crisis will not put deposits at risk. In this case, the bailout will only benefit the banker. On the opposite, in the case of a poor banker, who chooses the highest tolerated η, before the bailout, deposits would suffer losses, and the bailout would first be devoted to them. Consequently, the redistributive impact of an increase in the size of the bailout is not straightforward. Finally, and this will apply at all dates, and not only when crises occur, given the risk premium, a higher share of risky assets increases both the total income of bankers and its dispersion.

5.1. The commitment case

To assess quantitatively the impact of the absence of commitment, we build a model where the bailout rule, apprehended through the tax rate on labor income, $\overline{\tau}$, is constant. The program of the depositors and the bankers are respectively[15]:

$$
\begin{aligned}
V^D(a, s, A, \mathbb{1}_\chi) = \\
&\max_{c,a'} \Bigg\{ u(c) + \beta \Bigg[\\
&\quad \chi \Big(\pi^{dd} V^D(a'_\chi, s', A'_C, 1) + (1-\pi^{dd}) V^B(a'_\chi, s', A'_C, 1) \Big) \\
&\quad + (1-\chi) \Big(\pi^{dd} V^D(a', s', A', 0) + (1-\pi^{dd}) V^B(a', s', A', 0) \Big) \Bigg] \Bigg\} \\
&s.t.\ a' = (1+r^b)(a + sw(1 - \mathbb{1}_\chi * \overline{\tau}) - c) \\
&\quad a'_\chi = (1+r^b)(a + sw(1 - \mathbb{1}_\chi * \overline{\tau}) - c)(1 - \kappa(A)) \\
&\quad a' \geq 0
\end{aligned}
$$

[15] We have already assumed that the state of the economy could be restricted to the average financial wealth A, thus skipping the equivalent of programs (3) and (4) in this environment.

$$
\begin{aligned}
&V^B(a,s,A,\mathbb{1}_\chi) = \\
&\qquad \max_{c,a',\eta}\Bigg\{ u(c) + \beta\Bigg[\\
&\qquad\quad \chi\Big(\pi^{bb}\sum_k \pi^r_k V^B(a_{\chi,k},s',A'_C,1) + (1-\pi^{bb})\sum_k \pi^r_k V^D(a'_{\chi,k},s',A'_C,1)\Big) \\
&\qquad +(1-\chi)\Big(\pi^{bb}\sum_k \pi^r_k V^B(a'_k,s',A',0) + (1-\pi^{bb})\sum_k \pi^r_k V^D(a'_k,s',A',0)\Big)\Bigg]\Bigg\} \\
&\qquad s.t.\ a'_k = (1+\eta r^k + (1-\eta)r^a)(a + sw(1-\mathbb{1}_\chi * \overline{\tau}) + D - c) - (1+r^b)D \\
&\qquad\quad a'_{\chi,k} = max\left((a + sw(1-\mathbb{1}_\chi * \overline{\tau}) + D - c\right)\big[(1-\eta)(1+r^a) \\
&\qquad\quad +\eta(1-\varphi(1-\theta(A,\overline{\tau})))(1+r^k)\big] - (1+r^b)D; 0) \\
&\qquad\quad a' \geq 0 \\
&\qquad\quad \eta \leqslant= min\,(1,\eta^{max})
\end{aligned}
$$

with $\mathbb{1}_\chi = 1$ in case of a crisis.

The equilibrium definition is formally equivalent to that of sub-section 2.5, except that agents no longer need to anticipate next period's bailout as a function of the future state of the economy.

5.2. Comparing steady state equilibria

We here present the simulation results for different constant tax rates (the commitment case) and compare them with the baseline equilibrium.

We clearly see that the total financial wealth is decreasing with the tax rate. The higher is the tax rate, the larger is the proportion of losses which is bailed out, so the lower is the need for bankers to protect oneself's better. This effect is intuitive, at least from this viewpoint: agents expecting a larger financial loss will decide to accumulate more financial assets, among which a large proportion is risky. In other words, less bailout imply a higher volatility of the net return on risky assets. This, as we see, has the implication that the portfolio choice (the value of η) should be smaller, when the tax rate is lower, which is indeed the case.

Let us define the crisis-adjusted risk premium as the expected return on risky investment in excess of the safe return, taking the probability of a crisis into account, the magnitude of the bailout, and its impact on the net financial loss incurred by bankers. We simply compute the expected return on risky assets over time: by expected, we mean that we take both the idiosyncratic and the aggregate risks into account. In the absence of a crisis, the expected return on the risky asset is $E_0(\tilde{r}) = 3.75\%$. When a crisis occurs, we need to take into account both the gross loss φ and the bailout. The expected return on the risky asset, conditional on the occurrence of a crisis at the beginning of the current period, writes:

$$E(\tilde{r}_\chi) = E\left[(1 + E_0(\tilde{r}))(1 - \varphi(1 - \theta)) - 1\right]$$

In the above expression, θ depends on the state of the economy as it is hit by a crisis, and obviously on the exogenous tax rate.[16] The crisis-adjusted risk premium is the expectation over the realization -or not- of a crisis at the beginning of the next date:

$$RP_\chi = (1 - \chi)E_0(\tilde{r}) + \chi E(\tilde{r}_\chi)$$

When the constant tax rate is equal to its average value in the baseline model, 3.6%, (resp. with a constant tax rate of 0.8%, the crisis-adjusted risk premium is 3.07 (reps. 3.03). We see here that a lower bailout also reduce the risk premium, but only moderately. Besides, we see that the average loss on deposits, although of a much smaller order of magnitude, evolves qualitatively, like the average loss on bankers' wealth. Crisis are more costly for depositors too, which suggests that depositors may wish to save more (the evolution of the average beginning-of-period wealth of depositors[17] is consistent with this remark), in order to protect themselves against the occurrence of crises. Finally, we see that the evolution of the total financial wealth as the exogenous tax rate is reduced, is driven by several channels, going in different directions. The net impact is a significant increase in total wealth.

The welfare gains are computed as equivalent consumption variations, measured in percentage points[18], when comparing the welfare in a given scenario, with the baseline equilibrium. Agents would require an x percentage points increase of the average consumption in the baseline equilibrium (time-consistent equilibrium), to accept to remain in this environment, instead of moving to the considered scenario, when using the utilitarian criterion.

[16] Therefore, $E(\tilde{r}_\chi)$ is the mean net return over all bankers, as the weighted average of the mean net individual returns.

[17] It is also the case for the evolution of deposits.

[18] We convert the utilitarian criterion W into consumption $\hat{C}$ by assuming that the agent's consumption is constant: $W = \sum_{t \geqslant 0} \beta^t u(\hat{C})$

We see that when the government can commit to a given tax rate, the welfare is higher for a significantly lower tax rate, namely 0.8%. The welfare is itself computed over a long time period where crisis occur at random -the same time-period as that used to compute the various statistics. The welfare gains may seem rather small (0.05%), but one should bear in mind that we here compute an average welfare over a large number of periods, and crises occur only every 90 years. This method for computing the welfare may of course be subject to criticism -why take an average of welfare measured at different dates ?- but the alternative would be to compute it only for specific initial conditions. The choice of the initial state of the economy would be arbitrary, so, from our point of view, this computation does not seem more prone to criticism. What is remarkable is that, to achieve a 0.05% consumption gain, agents need be on average 5% richer. As savings earn an interest (0.5% for deposits as well as for the safe asset accessible to bankers, 3.75% for the expected risky return), part of the long-run gains come from higher capital income. Lowering the bailout does not seem very efficient. It would take a proper modeling of the transition between initial conditions typical of the baseline economy, to a lower bailout under commitment, to decide whether the long-run gains, measured in Table 6, are not out-weighted by transitional costs (during the transition, it is clear that agents will need to save more, if the total wealth is initially lower). We now turn to this specific point.

Tax rate (in %)	3.6 (t.c.)	0.7	**0.8**	0.9	1.3	1.8	3.6 (av.)	4.4
Avg. wealth	3.17	3.33	**3.33**	3.34	3.31	3.29	3.20	3.17
Avg. wealth (depositors)	2.16	2.26	**2.27**	2.27	2.25	2.24	2.18	2.16
Avg. wealth (banker)	1.02	1.06	**1.07**	1.07	1.06	1.05	1.02	1.02
Per Capita wealth, depositors	2.39	2.51	**2.52**	2.51	2.51	2.48	2.42	2.40
Per Capita wealth, bankers	10.15	10.63	**10.66**	10.66	10.59	10.51	10.24	10.15
Avg. Deposits	2.10	2.21	**2.22**	2.22	2.20	2.18	2.12	2.11
Av. loss on deposits (in %)	0.4	0.8	**0.7**	0.7	0.6	0.6	0.4	0.4
Av. loss bankers (in %)	28.3	36.2	**35.8**	35.4	34.1	32.6	28.5	27.1
Avg. η (in %)	63.0	62.6	**62.5**	62.5	62.6	62.6	62.9	62.9
Welfare gains (in % cons.)	0.0	0.04	**0.05**	0.05	0.04	0.03	-0.01	-0.03

Table 6: Aggregate statistics in the commitment model

5.3. *Modelling the transition between the time-consistent equilibrium and the long-run commitment optimum*

To go beyond long-run welfare comparisons, which are objectionable for the above-mentioned reason, we here ask the following question: assuming that the economy is currently characterized by the absence of commitment, with a government bailing banks out as she sees fit every time a crisis takes place, would the economy gain from switching to pure commitment ? From date $t = 0$, we set the crisis tax rate to a constant value ($\tau = \overline{\tau} = 0.8\%$), compute the transition of the economy toward the commitment equilibrium, and evaluate the welfare at date $t = 0$, as the commitment is just implemented. Comparing the $t = 0$ welfare for this transition, with the welfare in the absence of the switch to commitment, will inform us on whether enforcing commitment is worthwhile.

There are an infinity of different candidates for the initial distribution. We have chosen different initial conditions, all endogenously produced by the baseline model resolution, and differing with respect to the total wealth, in order to cover a broad range of situations. It needs be added, however, that initial conditions characterized by either high (around 4.0) or low (around 2.0) levels of total wealth, are quite rare, while intermediate levels of total wealth (around 3.0) are much more frequent.

Table 7 unambiguously answers, confirming the doubt which arose, that the long-run welfare gains seemed smaller, when compared with the required savings effort. Whatever the initial conditions are, it is always costly to switch to a commitment regime, characterized by a low level of bailout. There are several implications to this result. Costs are ordered according to what intuition would suggest: as the shock forces agents to save more, it is more costly, when it is implemented at times when wealth is lower. Quantitatively, costs are rather small, but the switch to commitment is nonetheless never beneficial, even when aggregate wealth is, by chance, very high.

Bailing out is costly for everyone, because of the tax rate increase, which is high, both because the government wishes to bail out a certain fraction of the incurred losses, and because the average share of risky assets is higher in this case (63.0% in the baseline case, versus 62.5% with a constant tax rate of 0.8%). Besides, bailing out mostly benefits bankers; the baseline bailout brings down the loss ratio for bankers to 28.3% (it is worth 35.8% with a 0.8% constant tax rate), while the loss ratio for depositors hardly diminishes (from 0.7% with a 0.8% tax rate, to 0.4% in the baseline case).

Initial total wealth	4.17	3.50	3.20	3.00	2.51	2.00
Welfare change (% consump.)	-0.03	-0.04	-0.04	-0.04	-0.06	-0.06

Table 7: Gains from switching to commitment

In Figure 3, we plot the transitional path of the total financial wealth for (i) the case where commitment is imposed at date $t = 0$ and (ii) the case where time-consistent choices still apply. It is clear from the graphs that the initial loss is higher when switching to commitment, as the bailout is lower. The first part of the simulation features frequent crises. We can see that, with a lower bailout, losses are larger, but only a few decades are necessary to catch up with the time-consistent bailout wealth. As soon as crises are less frequent, total financial wealth in the case of commitment exceeds its time-consistent counterpart.

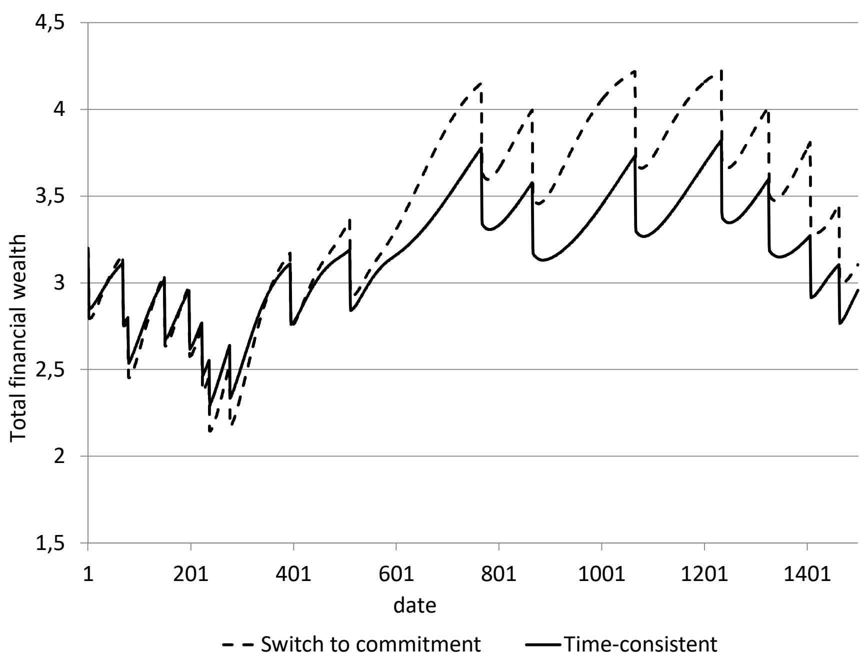


Figure 3: Dynamics of total financial wealth

6. Concluding remarks

In this article, we have built a heteregeneous-agents model of banking crises where depositor and banker agents differ with respect to the type of assets they have access to, and where the latter agents can additionally take advantage of depositors' savings as a leverage. The distributional properties reveal a high concentration of wealth, as one could expect with a significant risk premium. In our baseline economy, we assume away any commitment device and the bailout policy of the government is time-consistent. Bailout operations do not affect significantly the concentration of wealth among the richest agents. Finally, comparing the time-consistent equilibrium with constant bailouts, characteristic of commitment equilibria, shows that, in the long run, there are little welfare gains to be expected from a switch to commitment. More importantly, when taking the transition from the current time-consistent to the long-run commitment optimum, short-run costs in terms of reduced consumption dominate, so that such a change is not beneficial.

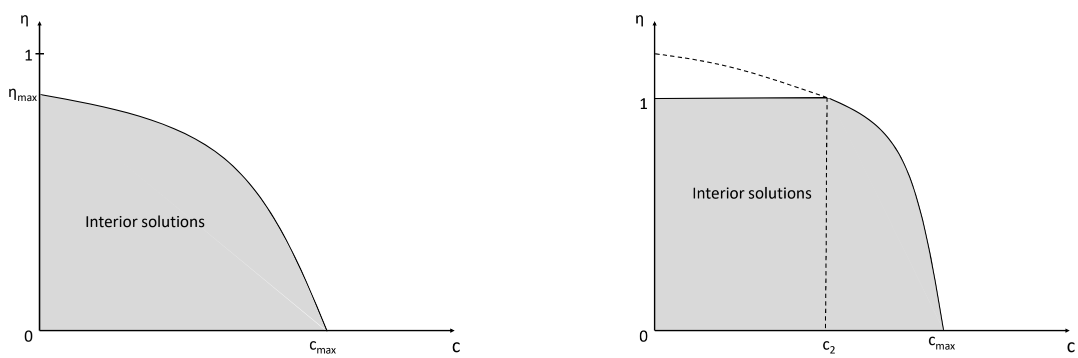
This model could be extended in several directions. First, the probability of a crisis could itself depend on the behavior of bankers and increase with the average risk taken by bankers. This would introduce a negative externality, each banker neglecting her own (small) impact on the global behavior of bankers. Secondly, borrowing could be allowed, either for depositors, or bankers, or both. The distributional properties and the welfare implications of time-consistent bailout choices could then be reassessed.

Appendix

Appendix A. Deriving the optimality conditions for bankers decision rules

The decision rules of the depositors are standard, as they only entail a savings/consumption rule of agents facing a borrowing constraint and a labor income risk (see Aiyagari (1994) for instance).

The behavior of bankers comprises both the global savings and the portfolio choice(η) decisions. Usual portfolio problems consider the constraint $0 \leqslant \eta \leqslant 1$, reflecting the impossibility for investors to sell short either of the two types of assets. This constraint is imposed here, to which we add the constraint that, in the absence of a crisis, depositors suffer no loss, no matter how unlucky the banker has been regarding her risky investment. Figures A.4 represent in a stylized way how this constraint restricts the set of choices $c(a, s, A, \tau), \eta(a, s, A, \tau)$, for both cases ($\eta_{max} \lesseqgtr 1$).



(a) Case $\eta^{max} < 1$

(b) Case $\eta^{max} > 1$

Figure A.4: Set of admissible choices for bankers

Left panel of Figure A.4 represents the situation in which the highest possible value for η, η_{max}, is smaller than one, which happens when the banker has a low stock of financial wealth. It could also be that η_{max}, as obtained from equation (1), is larger than one, which is plotted on the right panel of Figure A.4. In this case the graph is truncated to prevent η from being larger than one. The frontier will be denoted the $\mathcal{C}$ curve. This frontier is something original, but it is in fact the formulation of a usual constraint, the non-negativity of asset holdings, which, because of the leverage, makes the maximal admissible consumption C a function of the portfolio share η.

The program of the banker[19] can be formulated with Euler Equations and a Lagrangian, in order to have the various constraints appear in a straightforward manner.

$$\mathcal{L} = E_0 \left\{ \Sigma_{t\geqslant 0} \beta^t u(c_t) + \beta^t \lambda_t \left((1+\eta_t \tilde{r} + (1-\eta_t) r^a)(a_t + w s_t (1-\tau_t) + D_t - c_t) - (1+r^b) D_t - a_{t+1} \right) \right.$$
$$\left. + \beta^t \mu_t \left((1+\eta_t r^k_{min} + (1-\eta_t) r^a)(a_t + w s_t (1-\tau_t) + D_t - c_t) - (1+r^b) D_t \right) + \beta^t \nu_t (1-\eta_t) \right\}$$

The multipliers μ_t and ν_t are respectively associated to the non-negativity of next period's wealth even in the worst case and to the impossibility to borrow (short-selling of the safe asset). Multipliers associated with positivity of consumption and of the portfolio choice are not included[20].

The derivation with respect to c_0, a_1 and η_0 yields:

$$\frac{\partial \mathcal{L}}{\partial c_0} = 0 \Leftrightarrow u'(c_0) - E_0 \left[(1+\eta_0 \tilde{r} + (1-\eta_0) r^a) \lambda_0 \right] - \mu_0 (1+\eta_0 r^k_{min} + (1-\eta_0) r^a) = 0$$
$$\frac{\partial \mathcal{L}}{\partial a_1} = 0 \Leftrightarrow -\lambda_0 + E_0 \left[\beta \lambda_1 (1+\eta_0 \tilde{r} + (1-\eta_0) r^a) \right] + E_0 \left[\beta \mu_1 (1+\eta_1 r^k_{min} + (1-\eta_1) r^a) \right] = 0$$
$$\frac{\partial \mathcal{L}}{\partial \eta_0} = 0 \Leftrightarrow E_0 \left[(\tilde{r} - r^a)(a_0 + w s_0 (1-\tau_0) + D_0 - c_0) \lambda_0 \right]$$
$$+ \mu_0 (r^k_{min} - r^a)(a_0 + w s_0 (1-\tau_0) + D_0 - c_0) - \nu_0 = 0$$

This further simplifies as:

$$\begin{aligned} u'(c_0) &= E_0 \left[(1+\eta_0 \tilde{r} + (1-\eta_0) r^a) \lambda_0 \right] + \mu_0 (1+\eta_0 r^k_{min} + (1-\eta_0) r^a) \\ &= \beta E_0 \left[(1+\eta_0 \tilde{r} + (1-\eta_0) r^a) u'(c_1) \right] + \mu_0 (1+\eta_0 r^k_{min} + (1-\eta_0) r^a) \end{aligned}$$

[19] We do not introduce the possibility of the crisis and its implications here, because we only aim at showing what this new constraint modifies in terms of the model solution. The crisis, and its associated loss, does not affect by itself the resolution, apart from simply adding another shock. Contrary to the prudential rule, it does not add another potentially binding constraint.

[20] With a CRRA utility function and a risk aversion greater than 1, we know that as long as the after-tax labor income is strictly positive, the agent will always, by herself, choose strictly positive consumption levels, to avoid arbitrarily low levels of utility. Regarding the portfolio choice, it is a common result that, with a strictly positive risk premium, and with a labor income risk and a crisis risk independent from the individual choice in η, the agent will always choose strictly positive η, as the agent is risk neutral for extremely small risks.

$$\beta E_0 \left[(\tilde{r} - r^a)(a_0 + ws_0(1-\tau_0) + D_0 - c_0)u'(c_1) \right]$$
$$= \mu_0 (r^a - r^k_{min})(a_0 + ws_0(1-\tau_0) + D_0 - c_0) + \nu_0$$

Each of the two multipliers (μ_0 and ν_0) is either equal to zero, when the associated constraint is not binding, or strictly positive. We therefore have four possible cases:

- either both multipliers are equal to zero: the banker will choose by herself an interior solution in terms of portfolio and consumption;
- or $\mu_0 > 0$ and $\nu_0 = 0$: in this case, the prudential constraint is binding and the choice will be located on the $\mathcal{C}$ curve;
- or $\mu_0 > 0$ and $\nu_0 > 0$: the banker hits both constraints: $\eta_0 = 1$ and consumption is maximal given the prudential rule;
- or $\mu_0 = 0$ and $\nu_0 > 0$: the choice is interior in terms of consumption, but the banker hits the constraint $\eta_0 = 1$.

In the second case, we have:

$$u'(c_0) - \beta E_0 \left[(1 + \eta_0 \tilde{r} + (1-\eta_0) r^a) u'(c_1) \right] = \mu_0 (1 + \eta_0 r^k_{min} + (1-\eta_0) r^a)$$
$$\Rightarrow u'(c_0) - \beta E_0 \left[(1 + \eta_0 \tilde{r} + (1-\eta_0) r^a) u'(c_1) \right]$$
$$= (1 + \eta_0 r^k_{min} + (1-\eta_0) r^a) \frac{\beta E_0 \left[(\tilde{r} - r^a)(a_0 + ws_0(1-\tau_0) + D_0 - c_0)u'(c_1) \right]}{(r^a - r^k_{min})(a_0 + ws_0(1-\tau_0) + D_0 - c_0)}$$
$$\Leftrightarrow u'(c_0) - \beta E_0 \left[(1 + \eta_0 \tilde{r} + (1-\eta_0) r^a) u'(c_1) \right]$$
$$+ \frac{d\eta}{dc} \beta E_0 \left[(\tilde{r} - r^a)(a_0 + ws_0(1-\tau_0) + D_0 - c_0)u'(c_1) \right] = 0$$

where $\frac{d\eta}{dc} = \frac{-(1+\eta_0 r^k_{min} + (1-\eta_0) r^a)}{(r^a - r^k_{min})(a_0 + ws_0(1-\tau_0) + D_0 - c_0)} < 0$ is the derivative of the $\mathcal{C}$ curve.

Appendix B. Computation of the decision rules

The banker chooses c and η in order to maximize her expected intertemporal utility, as shown in program (6). The derivation with respect to the two choice variables, c_t and η_t, the result of which will be called residuals, are:

$$ResU'_c = u'(c_t) - \beta E(1 + \eta_t \tilde{r}_k + (1 - \eta_t) r_a) u'(\tilde{c}_{t+1})$$
$$ResU'_\eta = \beta E(\tilde{r}_k - r_a)(a_t + sw(1 - \tau_t) - c_t) u'(\tilde{c}_{t+1})$$

The first residual is simply equal to zero when the choice in consumption is interior, otherwise, it is strictly positive, with its expression provided in the previous appendix. The same applies to the second residual: either it is null when the choice in η is interior, or it is strictly positive.

First, one can show that, as we move along the $\mathcal{C}$ curve in an anti-clockwise fashion, the residual $ResU'_c$ strictly increases. Therefore, should there be a value for η such that $ResU'_c(c, \eta) = 0$ on the curve, it will be unique. For lower η (resp. higher η), $ResU'_c < 0$ (resp. $ResU'_c > 0$). Le us denote as η^* the value of η such that $ResU'_c(c^*, \eta^*) = 0$ with $(c^*; \eta^*)$ on the $\mathcal{C}$ curve. This implies that an interior solution in both η and c can only happen for $\eta < \eta^*$.

Secondly, for the optimal choice to lie on the $\mathcal{C}$ curve, we have proved in the previous appendix that the following condition holds:

$$ResU'_c + \frac{d\eta}{dc} ResU'_\eta = 0$$

One can show that the expression on the left-hand side is increasing, as we are moving along the $\mathcal{C}$ curve in an anti-clockwise fashion.

There are different cases, depending on the values of $ResU'_c$, $ResU'_\eta$ and $ResU'_c + \frac{d\eta}{dc} ResU'_\eta$. The algorithm to determine the precise case for each individual state (a, s, A, τ) is:

- if $ResU'_c(c_{max}, 0) > 0$ on the $\mathcal{C}$ curve:
  - if $ResU'_c(c_{max}, 0) + \frac{d\eta}{dc} ResU'_\eta(c_{max}, 0) > 0$, then the solution is $(c_{max}, 0)$;
  - if $ResU'_c(c_{max}, 0) + \frac{d\eta}{dc} ResU'_\eta(c_{max}, 0) \leqslant 0$, then search for the point (c_1, η_1) on the $\mathcal{C}$ curve where $ResU'_c(c_1, \eta_1) + \frac{d\eta}{dc} ResU'_\eta(c_1, \eta_1) = 0$ (or, $\eta = 1$ if $ResU'_c + \frac{d\eta}{dc} ResU'_\eta$ is the closest to 0 for $\eta = 1$).
- if $ResU'_c(c_{max}, 0) \leqslant 0$, then compute η_{max}.
  - if $\eta_{max} \geqslant 1$, compute $ResU'_c(c_2, 1)$ with $(c_2, 1)$ on the $\mathcal{C}$ curve.

* if $ResU'_c(c_2, 1) < 0$, compute, for $\eta = 1$, c_3 such that $ResU'_c(c_3, 1) = 0$. Compute $ResU'_\eta(c_3, 1)$.
  · if $ResU'_\eta(c_3, 1) \geqslant 0$, the solution is $(c_3, 1)$. Stop.
  · if $ResU'_\eta(c_3, 1) < 0$, find the interior solution (c_4, η_4) such that $ResU'_c(c_4, \eta_4) = 0$ and $ResU'_\eta(c_4, \eta_4) = 0$. Stop.
* if $ResU'_c(c_2, 1) \geqslant 0$, compute (c^*, η^*) on the $\mathcal{C}$ curve such that $ResU'_c(c^*, \eta^*) = 0$. Compute $ResU'_\eta(c^*, \eta^*)$.
  · if $ResU'_\eta(c^*, \eta^*) \leqslant 0$, find the interior solution (c_5, η_5) such that $ResU'_c(c_5, \eta_5) = 0$ and $ResU'_\eta(c_5, \eta_5) = 0$. Stop.
  · if $ResU'_\eta(c^*, \eta^*) > 0$, compute $ResU'_c(c_2, 1) + \frac{d\eta}{dc} ResU'_\eta(c_2, 1)$ for $(c_2, 1)$ on the $\mathcal{C}$ curve.

    1. if $ResU'_c(c_2, 1) + \frac{d\eta}{dc} ResU'_\eta(c_2, 1) < 0$, the solution is $(c_2, 1)$ on the $\mathcal{C}$ curve. Stop.
    2. if $ResU'_c(c_2, 1) + \frac{d\eta}{dc} ResU'_\eta(c_2, 1) \geqslant 0$, find (c_6, η_6) on the $\mathcal{C}$ curve with $\eta_6 \in [\eta^*; 1]$ such that $ResU'_c(c_6, 1) + \frac{d\eta}{dc} ResU'_\eta(c_6, 1) = 0$. Stop.

– if $\eta_{max} < 1$, compute (c^*, η^*) on the $\mathcal{C}$ curve such that $ResU'_c(c^*, \eta^*) = 0$. Compute $ResU'_{eta}(c^*, \eta^*)$.
  * if $ResU'_\eta(c^*, \eta^*) \leqslant 0$, find the interior solution (c_7, η_7) such that $ResU'_c(c_7, \eta_7) = 0$ and $ResU'_\eta(c_7, \eta_7) = 0$ with $\eta_7 \in [0; \eta^*]$. Stop.
  * if $ResU'_\eta(c^*, \eta^*) > 0$, find (c_8, η_8) on the $\mathcal{C}$ curve with $\eta_8 \in [\eta^*; 1]$ such that $ResU'_c(c_8, \eta_8) + \frac{d\eta}{dc} ResU'_\eta(c_8, \eta_8) = 0$. Stop.

Appendix C. Numerical implementation

In this section we discuss the implementation of the numerical algorithm used to find our main results.

Among the state variable characterizing the agent's program, the distribution of agents Ψ is a mathematical object of infinite dimension. For numerical purposes, and following Krusell and Smith (1998), we assume that the distribution of agents can be approximated by its moments, and we restrict to its first order one, that is, the average beginning of period financial wealth of agents A:

$$A_t = \sum_{i=D,B} \sum_{s \in S} \int_A a \Psi^i(a,s) da$$

The model is characterized by 4 state variables. Two state variables are individual and they track the individual wealth a_t and labor market status s_t of both bankers and depositors. The other two are aggregate variables: the above mentioned aggregate financial wealth A_t and the economy wide tax rate τ_t.

We discretize the AR(1) process governing the labor market outcomes into a 3-states Markov chain and use standard grid discretization techniques for the other state variables. The individual wealth grid is discretized using a non-uniform grid that is finer close to the lower bound on individual wealth and gets coarser as we move away from it. The grids on the aggregate variables are uniform.

In the function computing the individual decision rule of the depositor, we iterate over their Euler equation, using a root finding method to pin point the optimal savings for next period given the current states. The computation of the decision rule of the banker uses a similar approach but is complicated by their portfolio choice. In this step, given current state variables, we implement the algorithm described in Appendix B.

The implemented algorithm looks for the fixed point of the 4 following rules: (i) the law of motion of the aggregate financial wealth (both when a crisis is expected and otherwise) that relates the future value of the financial wealth A_{t+1} to the current aggregate states (A_t, τ_t); (ii) the law that determines deposits D_t given current aggregate states (A_t, τ_t); (iii) the law that determines the average loss on deposits κ_t given current aggregate states (A_t, τ_t);

The laws on the aggregate financial wealth are needed to forecast the future evolution of the economy. The other laws are necessary because although agents have an information on their individual deposits level and individual losses, they also need to have an idea of the aggregate counterparts of these variables, that are realized concurrently.

Thus our numerical strategy is the following:

1. We make a guess on the parameters entering the 4 aggregate laws,
2. Given our guesses, we solve individual consumption and saving/portfolio decisions over the whole state space to obtain the appropriate policy rules,
3. We simulate the economy in steps to obtain aggregate statistics. First given the above policy rule, we compute the stationary distribution in an economy were no crisis is

assumed. This yields a starting distribution that we use to simulate 100000 periods of the full economy. In each period, given the previous period aggregate financial wealth and tax rate as well as the distribution of agents and the policy rules, we are able to determine the next period distribution of agents. In some period and according to a random draw, a crisis can occur. In that case, the government determines the optimal bailout ratio, by considering a range of tax plans: for each tax rate to be implemented in the subsequent period, the ensuing bailout ratio is computed and the associated welfare is computed. The government then picks the tax rate that maximizes the computed welfare. The existence of a positive or a zero bailout ratio disturbs the distributions dynamics in each periods.

4. We discard the first 5000 simulations and use the remaining to generate time series for A_t, A_{t+1}, τ_t, D_t and κ_t. The time series are used to estimate the parameters of the five laws and their values are subsequently updated using a relaxation method to obtain new guesses.
5. Steps (2) through (4) are repeated until a fixed point on all the laws has been found.

The above strategy is implemented in the C language and appropriate sections are parallelized on the computer. As a benchmark, on an Intel Core i7 computer running at 2.8 GHz and 16 GB of RAM, and starting from an educated guess, the program runs in approximately one hour.

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