Time-consistent unemployment insurance

Sumudu Kankanamge*, *Toulouse School of Economics, University of Toulouse Capitole*,

Thomas Weitzenblum, GAINS-TEPP, Université du Maine and CEPREMAP.

Abstract

This paper examines the optimal time-consistent unemployment insurance policy in a search economy with incomplete markets. In a context of repeated choice without a commitment device, we show that the optimal replacement rate depends on how frequently in time the policy can be revised. The exact relation is dependent on the political process: if the utilitarian welfare criterion is used, the optimal rate is higher the shorter the choice periodicity. Self-insurance reduces the need for the public scheme but mostly because the policy cannot be changed often enough. The comparison with an economy where a commitment device is assumed shows that the commitment rate is close to time-consistent rates with very long choice periodicities.

JEL classification: J65, E61, C63.

1 Introduction

As one of the main policies available to mitigate the effects of unemployment, unemployment insurance (UI) programs have received a wide attention from the literature, focused mainly on the moral hazard and otherwise cost/benefits issues that arise[1]. In this paper we explore a much less documented question related to the UI policy: how would a benevolent social planner implement the UI policy in a dynamic setting when a commitment technology is not available[2] ? To assess this issue, we use a time-consistent environment

*Corresponding author. E-mail: sumudu.kankanamge@tse-fr.eu.

[1]Seminal papers on the subject include Shavell and Weiss (1979), Wang and Williamson (1996) or Hopenhayn and Nicolini (1997).

[2]In a world with a fully rational government, time-inconsistency issues appear with second-best policies as established by Fischer (1980), Hillier and Malcomson (1984) or Calvo and Obstfeld (1988). The seminal paper by Kydland and Prescott (1977) introduces the time-inconsistency principle much debated afterwards in the context of monetary policy although the issue is already exposed in Strotz (1955).

and detail the key mechanisms that impact equilibrium outcomes. Ultimately, we compare the time-consistent setting to the case where a commitment device is assumed.

We use a simple search model with incomplete markets as the basic building block of our economy: agents face an unemployment risk and engage in precautionary saving in order to smooth consumption. Once unemployed, agents receive a flat replacement income instead of the full wage and exert a search effort which determines the probability of finding a job. The benevolent social planner, otherwise called the government, provides public insurance through a tax-financed UI program and chooses the UI replacement rate. A key feature is the absence of a commitment technology: the government is unable to bind the economy to the currently implemented UI policy forever. Both the government and the agents expect that another government will find it optimal to set a new policy at some point in the future[3]. We use the utilitarian welfare criterion as the reference political choice mechanism but also consider the median voter case. Our main focus is the low-qualified workers segment, prime target of most UI programs, and we use the Current Population Survey and the Survey of Consumer Finances to match the characteristics of this subpopulation. As we omit part of the economy, we only describe its partial equilibrium.

There is an insurance and disincentive effect of increasing the UI replacement rate. Insurance benefits will accrue in the current and subsequent periods whereas consequences of the disincentive effect will appear only in the future as incentives to search are forward-looking. Absent a commitment device, the government has a temptation to deviate from past policies and provide more insurance now at the cost of only future higher unemployment. If self-insurance is possible, another reason to deviate appear: the government might want to immediately increase consumption by relaxing the precautionary saving motive through a higher replacement rate. Our results show that the temptation to deviate upward depends on the frequency of the policy choice: if the utilitarian criterion is the decision mechanism, it is stronger if the UI policy is revised more often and consequently higher replacement rates are implemented. Setting aside savings, this is explained by the fact that incentives costs increase at a higher rate than insurance benefits when the time between two policy decisions is increased[4]. When agents can save, additional mechanisms

[3]This can be viewed as the government playing a game against its future self with respect to the UI policy. Formally, we describe (subgame) Markov-perfect equilibria.

[4]In Kankanamge and Weitzenblum (2016), a directly related paper, we use a model without savings to show that marginal disincentive costs increase more than proportionally with the duration of the policy whereas marginal insurance gains increase proportionally. Moreover, we also use an analytical model to

appear. First, the optimal time-consistent replacement rate is lower with self-insurance: this is the traditional government vs. private insurance canal[5]. Additionally, if the disincentive cost is small, the government will implement higher replacement rates and help relax precautionary saving, but when the cost grows larger, it lets agents self-insure more. Thus when the policy is frequently revised, UI is prevalent whereas when it is revised less often, the government lets self-insurance increase in place of UI. The median voter case displays an opposite relation: UI replacement rate decrease when the policy is frequently revised. The median voter is typically employed and has a reasonable amount of savings: these elements are the main determinants of this result. Given these insights, our simulations show that the optimal time-consistent UI replacement rate, for a reference case where the policy is changed on average every 4 years, is 60.5%, using a US labor market calibration. The optimal rate increases sharply if the policy choice is more frequent. We provide a decomposition of the dynamics of the model and explain how exactly the unemployment rate and the accumulation behavior of agents enter in the determination of the time-consistent equilibrium. In terms of policy, this decomposition shows how the interplay between private savings and unemployment might prevent an actual government from making drastic adjustments of its UI policy and, all other things equal, how it might take time and several governments to reach an optimal policy when starting away from it. Finally, the comparison with an economy with commitment shows that the replacement rates in the commitment case are close to the time-consistent ones with very long policy revision frequencies.

This paper can be related to several strands of the literature. As stressed above, UI programs have been extensively analyzed, especially in the context of the principal-agent framework. Closer to this work, the role of UI policy in an incomplete markets setting has been first investigated in Hansen and İmrohoroğlu (1992). A substantial number of papers, among which Costain (1997), Acemoglu and Shimer (2000), Pallage and Zimmermann (2001), Wang and Williamson (2002), Joseph and Weitzenblum (2003) or Young (2004), have followed. Although our initial search environment with precautionary saving can be related to the latter papers, none of them consider the commitment issues at stake here. The time-consistent framework we use has early theoretical foundations in Cohen and Michel (1988) and the Markov-perfect equilibrium is formally defined in Maskin and Tirole (2001). Our equilibrium concept is mostly derived from Krusell et al. (1997). This

describe the existence of an additional redistribution mechanism.

[5]A first formal exposition with commitment issues can be found in Bruce and Waldman (1991).

concept has been used and refined in several subsequent papers as part of a recent effort to explore time-consistency policies: Klein and Rios-Rull (2003) quantitatively assess optimal fiscal policy in the absence of commitment, Krusell (2002) solves differentiable Markov equilibria in the context of redistribution policies, Klein et al. (2005) use a two-country economy with capital mobility to study optimal taxation without commitment and Klein et al. (2008) devise a compact characterization of the Markov-perfect equilibrium and applies it to the provision of public goods. The latter papers are either methodological or are not explicitly about the UI policy. However, as we are interested in optimal UI policy and associated political economy questions in a quantitative time-consistent setting, they are among the closest to the current work. Finally, the above mentioned Kankanamge and Weitzenblum (2016) is a directly related paper: both an analytical and a simulated model are used to characterize the time-consistent UI policy. However, the time-consistent framework used is much simpler and does not consider the important private versus public insurance considerations, the implications of the choice of the political decision mechanism or the introduction of a commitment device.

The rest of the paper is organized as follows. The next section presents our reference time-consistent model as well as our main results and the decomposition of the dynamics of the model. Section 3 presents the commitment case and compares it to the reference model. Section 4 concludes.

2 The time-consistent case

We describe the partial equilibrium in a Bewley (1986)-Huggett (1993)-Aiyagari (1994) type economy where a benevolent social planner, that we call government, sets the replacement rate of the unemployment insurance (UI) system in order to maximize social welfare. This behavior of the government is called policy and describes the political problem of choosing sequentially in time a current period optimal replacement rate. This problem relates to the one described in Krusell et al. (1997). The partial equilibrium will be characterized by (i) a law of motion for the economy and (ii) a choice rule. The former computes next period's level of aggregate financial asset and unemployment, given the current state of the economy. The latter associates, to any state of the economy, the replacement rate chosen by the government as anticipated by the agents.

2.1 Model specification

2.1.1 Households

The economy is populated by a continuum of *ex ante* identical infinitely lived households of unit mass. Their preferences, assumed to be additively separable over time, are summarized by:

$$\mathcal{V} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t v(c_t, s_t) \right\} \tag{1}$$

with:

$$v(c_t, s_t) = \frac{c^{1-\sigma}}{1-\sigma} - s^{\xi}, \ \xi > 1$$

where β is the discount factor, σ is the relative risk aversion, c is current consumption and s is the search effort of an unemployed agent. The search effort only influences the exit rate out of unemployment and thus does not apply to the employed agent who will trivially provide an effort $s = 0$. We note that an employed agent will have a status $\varepsilon = e$ on the labor market and the unemployed $\varepsilon = u$. The exit rate out of unemployment is assumed to be : $\pi(\varepsilon_{t+1} = e|\varepsilon_t = u) = 1 - exp(-\kappa s)$, with κ an exogenous parameter. Employed agents face a constant exogenous job destruction rate δ.

The budget constraint of a typical agent is :

$$\begin{aligned}
&a_{t+1} + c_t = (1+r)a_t + y(\varepsilon, \rho_t)(1 - \tau(\rho_t, U_t)) \\
&y(e, \rho_t) = w, \\
&y(u, \rho_t) = \rho_t w
\end{aligned}$$

where a_t is the agent's private financial asset level, r and w are exogenous prices, ρ is the replacement rate, U is the current unemployment rate and τ is the tax rate.

The recursive formulation of the household's program is:

$$
\begin{aligned}
& V\left(a, \varepsilon, \Psi, \rho\right) = \max_{c,s,a'} v(c,s) + \beta E V\left(a', \varepsilon', \Psi', \rho'\right) \qquad (2) \\
& \text{s.t.} \\
& a' + c = (1+r)a + y(\varepsilon, \rho)(1 - \tau(\rho, U)) \\
& \Psi' = \Gamma\left(\Psi, \rho\right) \\
& U = U\left(\Psi\right) \\
& \rho' = \Phi\left(\Psi'\right) \text{ with probability } \lambda \\
& \rho' = \rho \text{ with probability } (1-\lambda) \\
& y(e, \rho) = w \\
& y(u, \rho) = \rho w \\
& c \geq 0 \\
& a' \geq a_{min}
\end{aligned}
$$

Ψ (resp. Ψ') denotes the current (resp. future) measure of agents over asset holdings and employment status and Γ is the law of motion between two such consecutive measures. Φ is a function that describe the choice process of the government with respect to the replacement rate. λ is the probability that a new replacement rate is chosen at every date[6]. The optimal search effort s is such that the following equation is satisfied:

$$
\xi s^{\xi-1} = \kappa e^{-\kappa s}\left(\beta \mathbb{E}[V(a', e, \Psi', \rho') - V(a', u, \Psi', \rho')]\right)
$$

whereas optimal consumption is derived by the usual Euler equation.

2.1.2 The government

The government runs an unemployment insurance system and levies taxes to fund it. We already assumed that labor and replacement income were taxed at a proportional rate τ. Thus the government budget constraint is:

$$
\begin{aligned}
\rho_t w U_t &= \tau_t(w(1-U_t) + \rho_t w U_t) \\
\tau_t &= \frac{\rho_t U_t}{1 - U_t(1-\rho_t)}
\end{aligned} \qquad (3)
$$

[6] If $\lambda < 1$, the replacement rate ρ will, with some probability, last more than a single period, and thus needs to be kept as a state variable.

2.1.3 The dynamic game of the benevolent social planner

At the beginning of each date, the government chooses a new replacement rate with probability λ . This policy will only last until the next choice as no commitment technology is available. Thus the government can be seen as playing a game against its future self as it has no control over future choices. As future government choices are exogenous to both the agents and the government, they can be regarded as reaction functions. In the end, the social planner sets today the replacement rate for the subsequent period by maximizing the following utilitarian welfare criterion:

$$\Phi(\Psi) = \arg\max_{\tilde{\rho}} \sum_{\varepsilon \in \{e,u\}} \int_{a_{min}}^{a_{max}} V\left(a, \varepsilon, \Psi, \tilde{\rho}\right) \Psi(a, \varepsilon) da$$

2.2 The politico-economic equilibrium

The recursive equilibrium is characterized by the vector:

$$\left[a'\left(a, \varepsilon, \Psi, \rho\right), s\left(a, u, \Psi, \rho\right), V\left(a, \varepsilon, \Psi, \rho\right), \Gamma\left(\Psi, \rho\right), \Phi\left(\Psi\right)\right]$$

such that:

1. Given the law of motion $\Gamma\left(\Psi, \rho\right)$ and the choice rule $\Phi\left(\Psi\right)$, $V\left(a, \varepsilon, \Psi, \rho\right)$ is the value function solution to the program (2), $a'\left(a, \varepsilon, \Psi, \rho\right)$ the associated saving rule and $s\left(a, u, \Psi, \rho\right)$ the associated effort,

2. Given the rules $a'\left(a, \varepsilon, \Psi, \rho\right)$ and $s\left(a, u, \Psi, \rho\right)$, and for any state of the economy $\left(\Psi, \rho\right)$, next period's distribution of agents, Ψ', implied by the saving and effort rules, is consistent with the expected law of motion $\Gamma\left(\Psi, \rho\right)$,

3. Given the above value function, the maximization of the utilitarian criterion at each date is consistent with the expected choice rule $\Phi\left(\Psi\right)$.

Unfortunately, the characterization of the equilibrium above is not tractable because the distribution of agents belongs to a set of infinite dimension. To circumvent this difficulty, we follow the approach pioneered in Krusell and Smith (1998): we approximate the distribution of agents by the mean value of asset holdings. We detail the approximate aggregation in Appendix A.

2.3 Calibration

The benchmark calibration of the model corresponds to the US labor market. We use a model periodicity of three weeks to capture the quick flows on the US labor market. We focus on the segment of low-qualified workers as they are prone to be the main target of unemployment insurance policies. Precisely, we have to set values for $\sigma, \beta, \xi, \kappa, \delta, r$ and w. Without any loss of generality, the wage rate w can be normalized to 1. The relative risk aversion σ is set to a value of 2, which falls in the range of values usually admitted for this parameter, due to the wide range of estimates in the literature. We set the model interest rate to $r = 0\%$. This is a good approximation of the real return on the assets detained by the fraction of the population we are considering. For the most part their assets are low or no return bank accounts.

This model incorporates only a single saving motive, namely, the precautionary one while others such as retirement savings are not considered. Thus, it would be fair to expect that the average financial wealth would be quite low in the simulations. With income risk, the combined values of the discount factor β and the interest rate r determine the incentive to save. In turn, the average financial wealth held will affect the ability of agents to self-insure and, consequently, influence the optimal trade-off of the government. It follows that the quantitative results will, among others, depend on the quantification of β and r. We use data from the Survey of Consumer Finances (SCF) to compute the average liquid asset of low-skilled workers and thus obtain an empirically relevant calibration of the discount factor. In our computations using SCF (2007), the average per capita liquid financial asset of the fringe of the population we consider is 2926 \$. This is about 2.6 times the model-period income of said population[7]. This value is a target to pinpoint the value of the discount factor. We find a discount factor of $\beta = 0.9945$.

ξ, δ and κ directly affect the search intensity of the unemployed agents. Their calibration is therefore based on what we regard as the main quantitative properties of the US labor market for low-qualified workers. Precisely, we intend to reproduce (i) the unemployment rate for this type of workers, (ii) the average unemployment duration and (iii) the elasticity of the average unemployment duration with respect to the replacement rate. We use the Current Population Survey (CPS) to compute values for the first two elements. We retain CPS data from mid-2006 to mid-2007. We find an unemployment rate of about 7% for this fringe of the population. The unemployment duration is about

[7] Following the Bureau of Labor Statistics, the weekly first quartile income of high-school graduates and less is 376 \$. In model-period, this value becomes 1128 \$.

Parameter	Value	Description
w	1	Wage (normalization)
r	0%	Quarterly interest rate
β	0.9945	Discount factor
σ	2	Relative risk aversion
δ	0.0133	Job destruction rate
κ	0.2149	Exit rate parameter
ξ	1.9391	Curvature of effort function
a_{min}	0.0	Borrowing limit

Table 1: Benchmark calibration values

17 weeks in the data. Given the unemployment rate and the unemployment duration, it is straightforward to deduce the average employment duration, which in turn implies the job destruction rate. We obtain a destruction rate δ of 0.0133. Finally, the various estimations of the elasticity of unemployment duration with respect to the replacement rate do not establish a consensus. They suggest a positive value, below 1. In the US, we assume that the elasticity should be on the lower side of the plausible values and set a target value of 0.4. We find that with $\xi = 1.9391$ and $\kappa = 0.2149$, we match the above mentioned calibration targets.

This calibration has been implemented in a simpler model deprived of the political choice: an exogenous replacement rate is assumed to be held indefinitely, and the economy converges toward its long-run equilibrium. The baseline economy corresponds to a replacement rate equal to 40%. The elasticity of unemployment duration is computed by simulating the counterfactual experiment of marginally increasing the replacement rate, which yields its associated unemployment duration, and comparing it to its baseline counterpart.

2.4 Time-consistent replacement rate

The first and obvious effect of UI is the provision of insurance against the unemployment risk. The policy is implemented here through a tax financed flat replacement income available when unemployed and controlled with the UI replacement rate. Increasing the replacement rate improves insurance perspectives for the future and distributes additional revenues to the currently unemployed population. The UI policy also has a disincentive effect: the prospect of better insurance reduces the incentives to search for a job when

unemployed. These incentives are forward-looking: increasing the UI replacement rate today does not increase the current unemployment level. However, the prospect higher replacement rates in the future will increase future unemployment levels. Additionally, the public provision of UI also interacts with private insurance opportunities. Because agents have access to incomplete markets, they can partially insure against the unemployment risk on their own: there is a precautionary saving motive that encourages agents to self-insure. Faced with these elements and absent a commitment device, a government has a temptation to deviate towards higher UI replacement rates when deciding the policy. In a world without asset markets, the government might want to implement a higher replacement rate because the disincentive costs only appear in the future. In a world with incomplete markets, the government might additionally want to benefit from past saving efforts: increasing the UI replacement rate relaxes precautionary saving and increases consumption while still providing insurance. Our results quantify how exactly the government sets the UI replacement rate in a time-consistent framework with savings and how the temptation to deviate depend on the policy choice frequency. We also show that the results are dependent on the political decision mechanism. Finally we provide a detailed description of the dynamics of the model and argue why it has an importance for policy design.

2.4.1 Simulation results

The simulation results of the model described in section 2 highlight the time-consistency issue at stake here. They are reported in table 2. We consider alternative choice periodicities[8]: this is done by changing the value of the parameter λ. For simplicity, we express choice periodicity in years, so as to compare the behavior of a government that implements a new replacement rate on average every 4 years[9] to alternative choice periodicities. The specific results of the model in section 2 are under the label *Model with savings*[10].

The simulation results indicate that a positive and high replacement rate is sustainable in our economy. We can contrast the mechanisms taking place in a time-consistent environment with those in a long-run model (for instance in Hansen and İmrohoroğlu

[8]The choice periodicity defines how often we can expect a policy to be revised. For instance if the choice periodicity is 4 years, it is expected that a new governement enters office on average every 4 years and takes the decision to implement a new UI policy.

[9]This is our reference case to replicate the actual political cycle of many economies.

[10]Note that we also report the results of a different model where savings are precluded under the label *Model without savings* that we discuss later.

Choice periodicity (years)	0.058	0.25	0.33	0.5	1	**4**	10	25
Model with savings								
Replacement rate (%)	99.2	70.8	68.3	65.9	63.1	**60.5**	59.9	59.7
Unemp. rate (%)	51.2	10.7	10.1	9.7	9.3	**8.9**	8.8	8.8
Long run welfare (in % consump)	-43.42	-0.85	-0.56	-0.35	-0.14	**0.0**	0.03	0.04
Model without savings								
Replacement rate (%)	99.9	81.9	80.0	77.8	75.1	**72.5**	71.9	71.7
Unemp. rate (%)	91.2	10.7	10.4	10.1	9.8	**9.5**	9.4	9.4

Table 2: Characteristics of the time-consistent model

(1992)) or in models where a single transition between two steady-states is implemented (for instance in Young (2004)). In the current framework, the government re-optimizes over time and reconsiders the trade-offs it is facing at each choice. On the one hand, when starting from a high replacement rate and implementing a lower one, some long-run gains from higher asset accumulation and lower unemployment are expected. On the other hand, the government also optimizes over short run costs: less public insurance and the fact that a lower replacement rate prompts more self-insurance with a higher cost in terms of consumption. The new mechanism here is that agents' expectations endogenizes the fact that the implemented policy might be changed in the near or distant future. It is not obvious that long-run gains dominate. To illustrate this point, we provide a welfare measure of the time-consistent policy in the last row of table 2. Precisely, this is a measure of the long-run consumption gain/cost of imposing the optimal time-consistent replacement rates reported by our simulations instead of maintaining a reference average replacement rate. We set the reference to the replacement rate found in the time-consistent case where the government implements a new policy every 4 years. The long-run welfare measures indicate that none of the time-consistent policies above would have been implemented based on this criterion: higher replacements rates than the one found for a choice periodicity of 4 years reduces long-run welfare whereas lower rates increases welfare. The optimal rate based on long-run welfare would be a lower rate beyond the scope of the time-consistent rates reported here, even for the longest periodicity considered[11]. This is explained by the fact that this welfare criterion only considers the long-run (steady state) outcomes and for instance does not take into account the immediate costs to reducing the UI policy.

[11]The long-run optimal rate in the corresponding model is in fact 46%.

The simulation results further show that both the optimal time-consistent replacement rate and the unemployment rate unambiguously and monotonically increase as the choice periodicity becomes shorter. The longest choice periodicity we consider is 25 years ($\lambda = 0.0023$), and in that case we find the lowest replacement rate that the model produces for our range of simulations with a value of 59.7% and an associated unemployment rate of 8.8%. At the other end, we consider a case where the parameter λ is as close as possible to unity, which would be the behavior of a government choosing every period. Precisely, we simulate $\lambda = 0.9975$ which is equivalent to a periodicity of 3.12 weeks, very close to the periodicity of the model[12]. In that case the economy does sustain an extremely high replacement rate of 99.2%. The unemployment rate is also excessively high at 51.2%. This last result is theoretical and obviously disconnected from reality: it is explained by the fact that labor has no social value in this model. For the reference case of a choice every 4 years, the replacement rate is at 60.5% with an unemployment rate of 8.9%.

The fact that the value of the replacement rate is higher the shorter the choice periodicity can be explained by a combination of mechanisms. A first effect comes from the fact that the search effort of unemployed agents is purely forward looking: the decision to search more or less today is only impacted by the value of the replacement rate in the future. Thus if a new (higher) replacement rate is implemented today, this comes as a surprise to all agents and this policy will not change current search effort. Rather it will create what can be called a pure redistribution effect as it is not anticipated. This effect will encourage the government to pick a higher replacement rate in order to profit from the beneficial outcome. Second, there are the anticipated gains in terms of insurance and costs in terms of incentives to search. In total, the anticipated costs and gains must counterbalance the redistributive gains as otherwise the government would implement indefinitely high or low replacement rates. A central element is that these anticipated costs and gains depend on the choice periodicity. In fact, even in the absence of any redistributive gains, the fact that anticipated costs and gains are dependent on the choice periodicities play an important role in our results. It can be shown that gains and costs of a replacement policy does not change similarly as a function of the duration of the policy: when the choice periodicity becomes shorter, the costs of the policy decreases faster than the gains, making it optimal to implement a higher replacement rate. Thus

[12]We are unable to simulate exactly $\lambda = 1$. In this model, when a choice is made almost every period, the replacement rate is infinitely close to 100%, and the economy shuts down. However the adjustement of the unemployement rate is slower.

even if we totally rule out the pure redistribution effect, the time-consistent rate will be higher the shorter the choice periodicity[13]. In Kankanamge and Weitzenblum (2016), in direct relation to this issue, we use a simplified model without savings to show how gains and costs change with respect to the policy duration and detail the effects of getting rid of the redistributive effect.

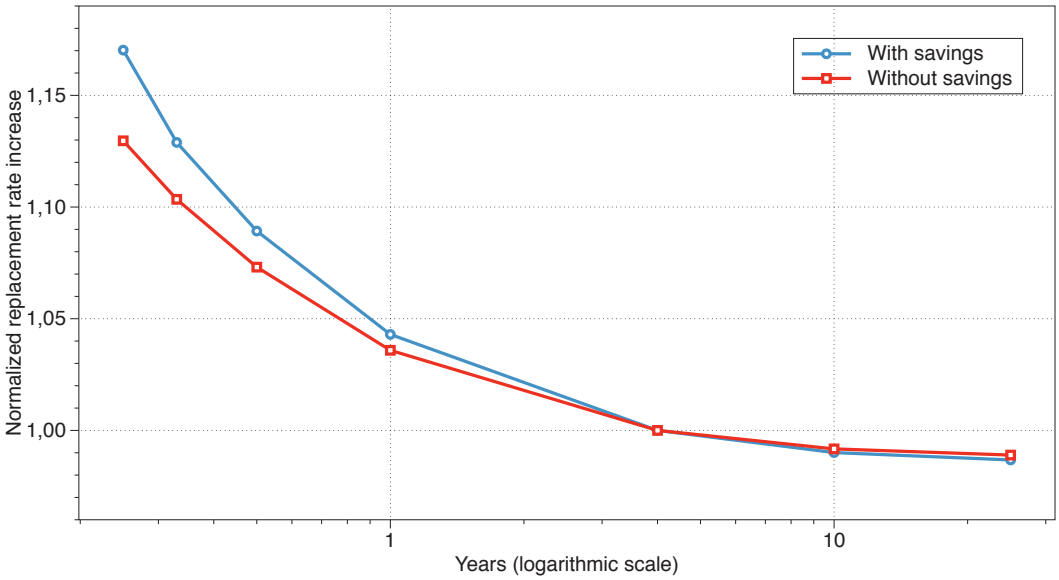

Figure 1: Relative replacement rate differences.

Another key result we underline is the importance of savings. To this end, we compare our results to a model where savings are precluded and report the results in table 2 under the label *Model without savings*. This model is virtually the same as the model described in section 2 but households do not have access to any assets and consume all their income. In table 2, one obvious difference we remark about these two models is that the optimal time-consistent replacement rate is always higher in the model without savings. This extends to the unemployment rate as it is also always higher in the model without savings. This difference can be explained by the traditional private versus government insurance channel. In a world without private savings, the government-run UI system provides the only means of insurance against unemployment risk. However once agents have access to an (even incomplete) asset market, they will engage in precautionary saving in order

[13]To rule out the redistribution effect, one can build a model where the policy is announced at period t but only implemented at period $t+1$ so that there is no more surprise and only anticipated effects are present. Moreover, we give a quantitative assement of the disincentive effect when we analyze the dynamics of the model below.

to smooth consumption and therefore will increase their insurance against unemployment risk on their own. This will automatically reduce the need for government insurance. In table 2, we can see that the replacement rate gap due to savings is quite significant at all choice periodicities: if we exclude the extreme case where the government changes the replacement rate every period, it amounts to a difference between 11 and 12 percentage points.

We also highlight another implication of savings that depend on the choice periodicity. To see it, we plot on figure 1 the replacement rates of table 2 in a way that shows how fast the replacement rates increase when the choice periodicities are reduced[14]. The UI replacement rates in both the saving and the no saving model have been divided by the respective replacement rate when the choice periodicity is 4 years, so as to normalize both curve to 1 when a choice is made every 4 years. The plot shows that for longer choice periodicities the points are very close but as the choice periodicity becomes shorter, they diverge and are higher in the model with savings: the government increase UI in a more substantial way in the model with savings as the choice periodicity becomes shorter. In both the model with savings and without savings, the disincentive cost decreases as the choice periodicity becomes shorter. However, the model with savings has the additional precautionary saving motive and the cost in terms of consumption it induces. If the choice periodicities are long, the disincentive costs are high, the government does not want to increase UI and self-insurance plays an important role. However, when choice periodicities are short and the disincentive cost decreases, the government is willing to increase UI also to reduce the consumption cost of self-insurance as evidenced by the figure. The choice periodicity is an important determinant of the mix between private insurance and public insurance in the economy. Given that the utilitarian criterion is the choice mechanism, if the UI policy could be revised more often, the government would provide generous UI and reduce the need for self-insurance.

2.4.2 Median voter

The benchmark model has the utilitarian criterion at the heart of the choice process but alternative ways to select the policy choice might alter our previous results. We underline here the traditional but striking case of the median voter. Alternatively to the utilitarian

[14]We exclude the case where the replacement rate is almost 100% to focus on the less exceptional cases even though including this point would show our point even more strikingly. Moreover, to emphasize the result, we use a logarithmic scale for the years on the horizontal axis.

approach, we now use the following criterion to set the replacement rate:

$$\Phi(\Psi) = \arg\max_{\tilde{\rho}} V\left(a_{med}, \varepsilon, \Psi, \tilde{\rho}\right)$$

a_{med} is the level of financial wealth such that 50% of the population is employed and has at least a level of asset of a_{med}[15]. The replacement rate chosen by this individual then applies to the economy.

Choice periodicity (years)	1	4	10	25
Replacement rate	43.2	55.0	57.0	57.6
Unemp. rate	7.2	8.3	8.5	8.6

Table 3: Characteristics of the time-consistent model with median voter

The results here are driven by the fact that the median voter in this economy is employed and comparatively rich. Thus her incentives are different from what we emphasized with the utilitarian welfare criterion. We first remark that the economy still sustains positive replacement rates but they are lower than those of the benchmark model. Second we note that the choice periodicity is once again an important determinant of the results. However we do not find in this case that the shorter the choice periodicity the higher the replacement rate, on the contrary: the optimal time-consistent replacement rate now monotonically increase the longer the choice periodicity and so does the unemployment rate.

These results are quite intuitive from the point of view of the median voter: if the choice periodicity is short, she has a strong probability of remaining employed between two policy changes. Thus she is not willing to pay for an increase in the replacement rate that she will most likely not benefit from. Thus in the short run she favors lower replacement rates. If the choice periodicity is long however, the median voter is aware that with a higher probability her status on the labor market might change and that she might become unemployed, between two policy changes. This leads her to be in favor of a

[15] It is quite intuitive, and we check it on the obtained value functions, that for a given employment status, the most preferred replacement rate is a decreasing function of the current level of asset holdings. Provided (also checked ex post) that all unemployed agents, no matter how rich they may be at the equilibrium, always favor replacement rates higher than that of the poorest employed, it is straightforward to notice that an employed agent, with half of the total population richer than her, will be median in terms of the replacement rate choice: all those employed and richer than her will favor lower replacement rates. Conversely, all the employed poorer than her, as well as all the unemployed, will favor higher replacement rates.

higher replacement rate than in the short run. The previous pure redistributive effect that we describe in the case of the utilitarian criterion will have quite a different impact here: indeed any surprise increase in the replacement rate is accompanied by a tax increase. The median voter does not currently benefit from the replacement rate increase but pays the tax immediately as everyone else. Thus the beneficial aspects of a replacement rate increase comes from anticipated factors that we underline above and not the surprise.

Our computations show that although the replacement rate remains at a steady level around 55% for medium to long choice periodicities, it falls sharply for shorter choice periodicities when the probability of the median voter remaining employed between two policy changes is realistically high. For instance, for a choice periodicity of one year, the replacement rate is as low as 43.2%. Moreover, the replacement rate in the median voter case for long choice periodicity is not very different from the utilitarian case. Thus the utilitarian and the median voter views are similar concerning policies that change less frequently in time: this is intuitive because when policies are revised after long periods of time, the unemployment perspectives as seen from today bears similar probabilities across agents.

2.4.3 Model dynamics

In this section, we return to the benchmark utilitarian case to describe in more details the dynamics and the mechanics of the model and derive economic insights on the choice process of the government.

We start by reporting in table 4, both for our reference choice periodicity of 4 years and a shorter periodicity of 1 year, the computed coefficients values of the laws of motion of the average financial asset and the unemployment rate as well as the choice rule[16]. The interpretation of these values and the way they change with respect to the choice periodicity provides a deeper understanding of the mechanics of the model. Focusing our attention first on the law of motion of the unemployment rate, the most interesting coefficient is α_3^U, that measures the disincentive effect. For both choice periodicities, it enters with a positive sign: a higher replacement rate today reduces search efforts and lead to more unemployment tomorrow. It is clear that when the choice periodicity is shorter, this coefficient is smaller: when the government is expected to revise its UI policy soon, one can count on a current replacement rate increase only for a limited time and this should induce agents to search for a job with more effort comparatively to a case

[16]These laws are explicitely explained in Appendix A, as well as the signification of each coefficient.

Choice periodicity (years)	Parameter values			
Law of motion: unemployment rate				
	α_0^U	α_1^U	α_2^U	α_3^U
1	0.001271	0.000354	0.861374	0.017940
4	-0.000493	0.000562	0.851999	0.021743
Law of motion: aggregate financial asset				
	α_0^A	α_1^A	α_2^A	α_3^A
1	0.162460	0.959571	-0.649952	-0.110052
4	0.185707	0.960456	-0.603276	-0.154332
Choice rule				
	α_0^ρ	α_1^ρ	α_2^ρ	
1	0.594323	-0.043710	0.771676	
4	0.590248	-0.024293	0.425752	

Table 4: Parameter values for the law of motion of average financial wealth

where the choice periodicity is longer. As a consequence, a current replacement rate increase should augment next period's unemployment less. This coefficient thus gives a quantitative assessment of the disincentive effect. We can add that comparing all choice periodicities we simulate, the reduction of this coefficient is more and more striking as choice periodicities become shorter: it is around 0.014 for a choice periodicity of 6 months and about 0.011 for 4 months. However, its change is almost not perceptible between longer choice periodicities. This is in line with the arguments we have previsouly used in the paper. Coefficient α_2^U is evidence of the persistence of this law of motion: the current unemployment rate is the main predictor of the next period unemployment rate. We also remark that the level of aggregate financial asset has a negligible impact on this law of motion as evidenced by parameter α_1^U. As a robustness exercise, we check later how our results change when we omit the average asset level. Next, turning to the law of motion of the aggregate financial asset, a first remark concerns the persistence of the law as evidenced by the parameter α_1^A. Also, both the current unemployment rate and replacement rate enter with negative signs. The negative sign on α_2^A, that measures the impact of current unemployment on future average asset, can be explained, all other things equal, by the fact that more unemployment forces unemployed into dissaving leading to the negative link. Turning to α_3^A, we have a measure of the private versus government insurance canal as this parameter quantify the impact of the current replacement rate on the future asset level. Contrastingly to the traditional private versus government insurance

canal measured in stationary models, this model and this coefficient provides a finer and dynamic measure of this relation. If the replacement rate is high today, there is less need for precautionary savings, agents will save less and the future asset level will be lower. Furthermore, if the replacement rate is high today and the agents expect this policy to change only in a distant future, they will save even less as they expect government insurance to be around for a longer time, thus the future asset level is expected to be even lower as evidence by the stronger negative link when the choice periodicity is 4 years. Finally, concerning the choice rule, α_1^ρ has a negative sign. This parameter measures how current assets impact the choice of the current replacement rate. If there is some level of financial assets in the economy, agents are self-insured to some extent and it is easier for the government to implement a lower replacement rate. But if the government implements the next policy in a distant future, this effect is less strong: even if agents are well insured privately, if the government cannot change the replacement rate in the near future, the private cost of maintaining precautionary savings for a long time has to be taken into account. The positive sign on α_2^ρ seems to only capture the purely redistributive effect at first: if the unemployment rate is very high today, the government has a strong incentive to increase the replacement rate and ease the burden on the unemployed as negative unemployment implications of such a policy will only appear in the future. However, the government is less inclined to do so if the policy cannot be undone in a near future as evidenced by the lower value of the parameter for the longer choice periodicity: this well illustrates the trade-off the government is facing between immediate gains and long term costs and how the purely redistributive effect is countered by anticipated elements if the choice takes place less often. We can add, for both parameters α_1^ρ and α_2^ρ that, the longer the choice periodicity, the more the specific aspects of the economy in the future is important and consequently the less the current (initial) aspects of the economy matter. This is a more general way of understanding why the values of these coefficients are smaller in absolute terms for longer choice periodicities.

We continue to analyze the dynamics of the model by detailing additional aspects of this framework: the determination of the equilibrium and the dynamics around this equilibrium. We start by briefly describing the simulation phase of the model. It is essentially a two steps procedure. First, using a stationary distribution as a starting point, government choices are simulated and an equilibrium replacement rate is computed. Then, in a second step, several trajectories are simulated around the equilibrium in order to capture the dynamic behavior of the economy and obtain time series to update the choice rule and the laws

of motion. To be precise, in this last step, starting from the same initial distribution, we define, at $t = 0$, ad hoc exogenous deviations of the replacement rate of various magnitudes. For each of these deviations, several time series are simulated in which the government randomly chooses a new replacement rate (or retain a previously chosen one) according to the probability λ. This step, one of the possible ways to assess the dynamics around the equilibrium[17], insures that we are indeed at a time consistent equilibrium as by construction no deviations from it should be optimal.

In figure 2 (bottom panel), we plot simulated trajectories around the equilibrium that correspond to the final simulation step described above and for the reference case of a choice periodicity of 4 years. We choose 3 specific trajectories, all starting from a same initial deviation, 1.4 percentage points below the equilibrium replacement rate. When selecting these trajectories, we purposely handpicked one trajectory where the first government choice is almost immediate (Simulation 1 where first choice takes place at period 8) and one where it is quite late (Simulation 3 where first choice takes place at period 362). The last trajectory (Simulation 2 where first choice takes place at period 83) is between the first two. We note that the first choice made by the government sets the replacement rate fairly close to its equilibrium value in all 3 simulations. However, the longer the government has to wait before making the first choice and the further is the implemented rate from the equilibrium rate. To show that this property is general and symmetric in our model whether starting from a deviation below or above the equilibrium rate, we use figure 3. In this figure, we plot all (but only) the first choices the government makes in the last step of our simulation process, reordered by increasing period of first choice. Each of the curves in this plot is an initial deviation away from the equilibrium rate of a given magnitude, from 2 percentage points above to 2 percentage below this rate. All curves show that the longer the government has to wait for its first choice and the further away this choice is from equilibrium. Additionally, this figure shows that after some periods have passed without making a first choice, there seems to be a threshold: for instance a first choice made after 100 periods will not be very different from a first choice made after 300 periods. However, first choices made for instance before 50 periods are quite different from each other and from first choices made later as there is much curvature in the left part of the horizontal axis. This is explained by the interplay between the accumulation behavior of agents and the evolution of the unemployment rate, as detailed

[17]We think that these marginal deviations are a natural way to assess this type of equilibrium as they are actual policies that a government might consider.

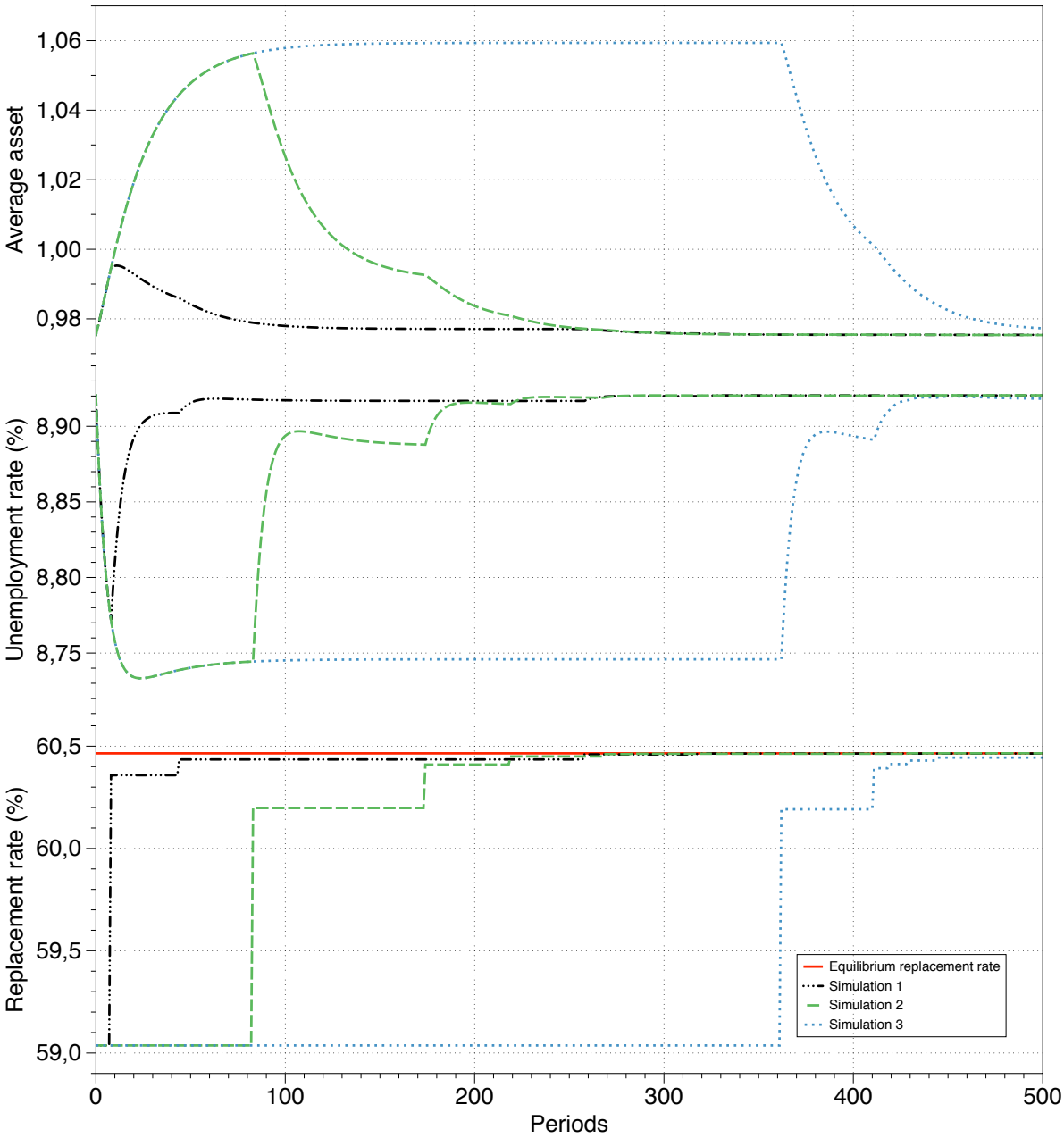

Figure 2: Simulated trajectories around equilibrium.

below.

These observations tend to show that the time varying elements in the model have some importance. The elements that change between government choices are ultimately the unemployment rate and the accumulation behavior (or more precisely the accumulated financial wealth) of agents. We reproduce the unemployment rate and aggregate asset paths for each of the simulated trajectories on the middle and top panel of figure 2.

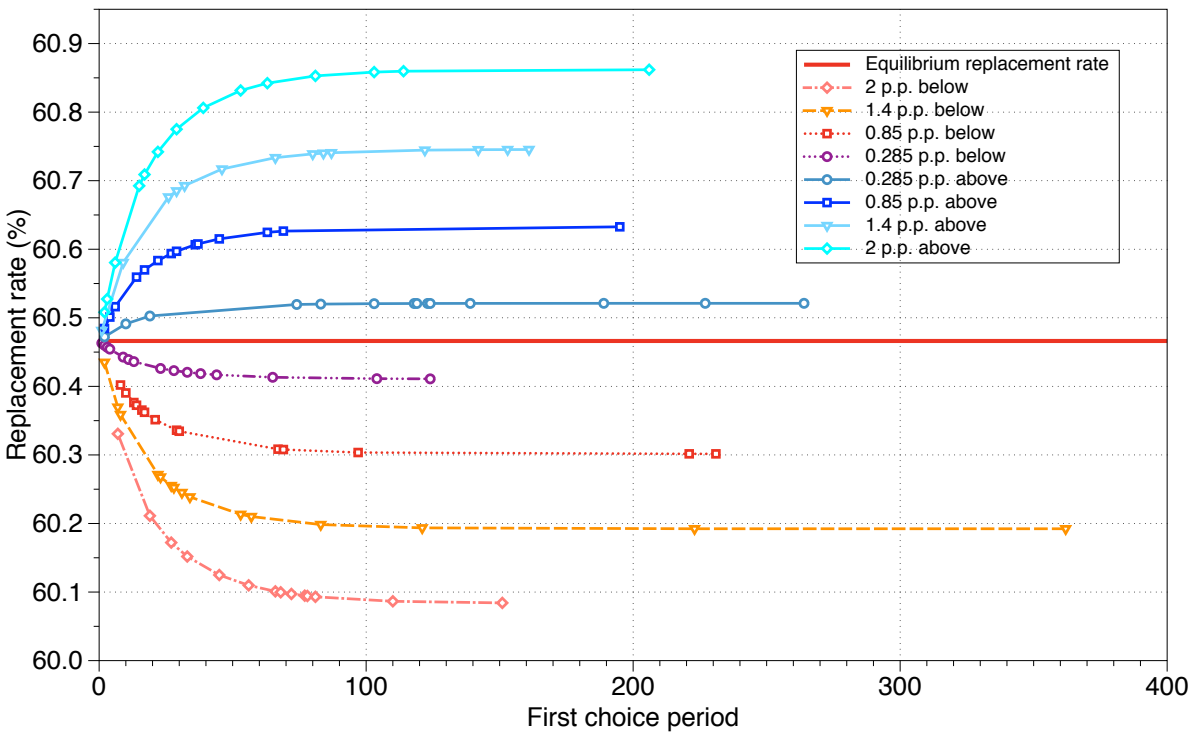

Figure 3: Simulated government first choices.

Regarding the unemployment rate, given an initial replacement rate below the equilibrium rate, if the first choice of the government arrives late, agents will exert a higher search effort than at the equilibrium rate, as long as the first choice is not implemented. In the meantime, the unemployment rate will decrease and the distribution of agents will slowly switch to a state less favorable to a replacement rate increase. When the first choice of the government is finally made, the conditions are thus no longer in place to choose a replacement rate very close to the equilibrium contrary to if this choice was made before. About the accumulation process, starting from the same initial point, a lower replacement rate encourages self-insurance through precautionary saving. The later the government's first choice takes place and the wealthier the agents are (which translate into better self-insurance). The accumulation behavior thus alters the distribution of agents and makes the situation less favorable to a replacement rate increase. Therefore the government will again not implement a replacement rate as close to the equilibrium in this situation as if the choice was made earlier. We have symmetric mechanisms when starting from an initial point above the equilibrium rate. In that case, agents will exert a lower search effort, the unemployment will rise and at the same time agents will save

less and their self-insurance will decrease. The alteration of the distribution of agents is such that the government will not implement a replacement rate decrease as close to the equilibrium as if the choice could have been made earlier. Finally, we remark that given an initial replacement rate, the unemployment does not evolve monotonously as the average asset adjustment does. For instance, in the case of simulation 2 and 3, before the first choice of the government, we observe that the unemployment rate decreases at first and then increases before stabilizing. This overshooting characteristic of the unemployment rate adjustment can be observed in all the simulated trajectories. This can be explained by the interaction between search effort (ultimately unemployment) and precautionary saving adjustments: subsequent to a replacement rate decrease, asset accumulation has to be changed in order to reach the new desired self-insurance state. However, it takes time to accumulate assets while the unemployment adjusts quickly. Thus until self-insurance is high enough, the unemployment rate decrease. When enough assets has been accumulated, the unemployment rate increases again.

We can additionally show some evidence of the fact that the accumulation behavior of households plays a larger role in shaping the above dynamics than the unemployment rate. To this end, we return to the comparison with the model where saving is precluded. In table 5, we simulate exactly the same trajectories as in figure 2, in both the model with and without savings and we compute the percentage of the distance to equilibrium covered at each choice. To be precise, each of the models yields a different equilibrium replacement rate, and the deviations we make during simulation are relative to said equilibrium. Thus, even though the distance from a given deviation to the equilibrium is imposed to be the same in both models, the values of the replacement rate at the deviation are different. As a consequence, we compute in each model the distance between the replacement rate implemented at each choice and the rate at the initial deviation as a fraction (in percentage) of the distance between the equilibrium replacement rate and the rate at the same initial deviation. In other words, given a simulated trajectory where choices take place at the same time in both models, the percentages in table 5 indicate how much of the distance between a deviation and the equilibrium the government has chosen to cover when setting the new replacement rate at each of the choices. For instance, in Simulation 1, the first choice of the government, that happens at period 8, covers 92.48% of the distance to equilibrium whereas, in Simulation 2 where the first choice happens at periods 362, it only covers 80.92%, in the model with savings. These numbers quantify some of the results presented above. But the most interesting point here is the comparison

with the model without savings. This model exhibits qualitatively the same behavior as the model with savings, however at every choice considered, the government decides to set a replacement rate much closer to the equilibrium rate than in its counterpart. In addition, the later the first choice of the government arrives and the larger the difference. Remembering that in the model without saving, only the search effort component and ultimately the unemployment dynamics are present, the accumulation behavior of agents in the model with savings is the major cause that explains why the government does not implement abrupt returns to the equilibrium rate.

Choice	Simulation 1		Simulation 2		Simulation 3	
	With saving	Without saving	With saving	Without saving	With saving	Without saving
1st choice	92.48	98.01	80.82	97.19	81.25	97.19
2nd choice	97.88	99.96	94.86	99.95	96.13	99.95
3rd choice	99.59	100.00	96.31	99.98	98.91	100.00
4th choice	99.90	100.00	97.50	100.00	99.70	100.00
5th choice	99.98	100.00	98.51	100.00	99.83	100.00

Table 5: Percentage of the distance to equilibrium covered at each choice.

The model dynamics we extract from our simulated cases are interesting because they can help explain the decision process in an actual economy that would be away from its optimal rate. If we match, for simplicity, the actual political decision process to about a choice every 4 years, these dynamics explain how the unemployment rate and accumulation behavior of agents interact and why it might not be optimal at that time for the actual policy maker to take drastic measures to return to the equilibrium rate. All other things equal, it might take several governments and a certain amount of time to reach the optimal UI rate.

2.4.4 Accuracy and robustness

In this section, we subject the benchmark model to several tests to validate our results. We start by looking at the accuracy of our laws of motion and rules. In appendix C, we conduct basic statistical fit tests using the coefficient of determination R^2 and an estimate of the error standard deviation $\hat{\sigma}$. According to these statistical tools, the fit is very good. However, given the nature of our model, the results depend on how well agents are able to predict the choice of the government: we need to determine how close the replacement rate predicted by the choice rule is to the effective replacement rate implemented by the

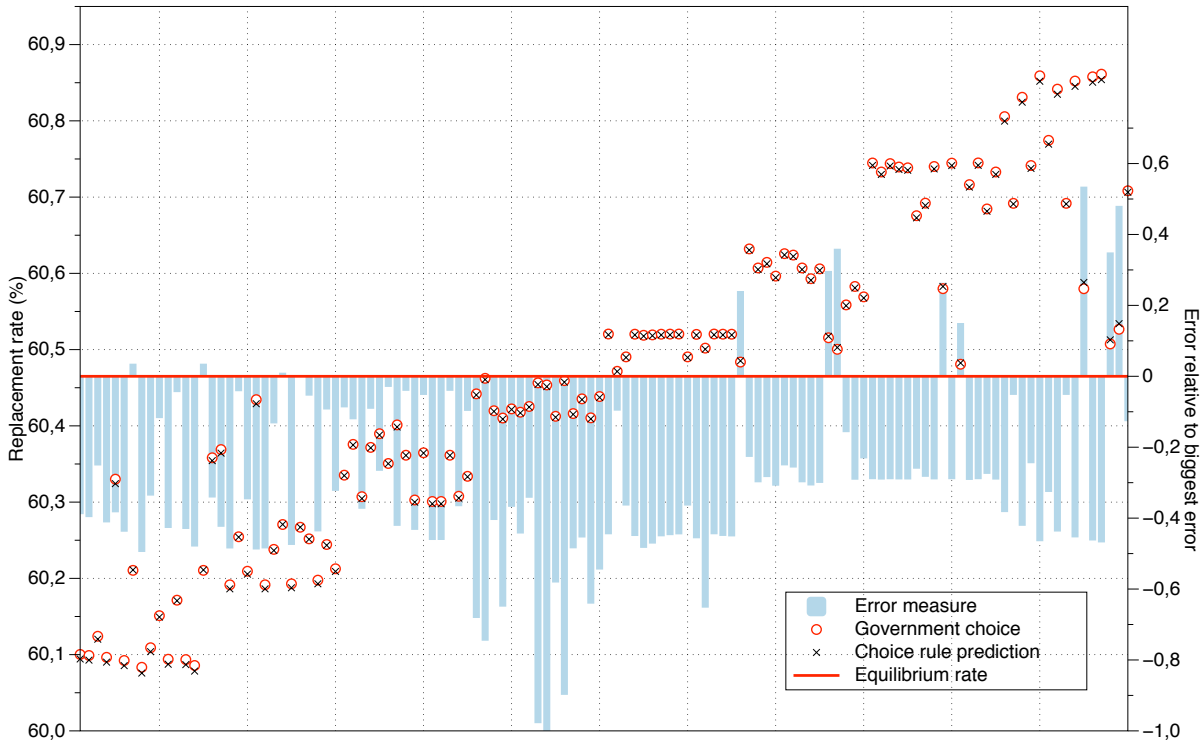

Figure 4: Government choices vs. choice rule forecasts.

government. As discussed before, given an equilibrium replacement rate and a deviation away from this equilibrium, the ability to correctly predict the first choice of the government is essential. To assess this error, we first compute the absolute difference between a replacement rate predicted by the choice rule and the replacement rate effectively implemented by the government, for all of the first choices[18] in our simulation step, given the equilibrium replacement rate and the deviations away from it. Because this absolute difference needs to be scaled, we divide it by the difference between the equilibrium replacement rate and the appropriate initial deviation considered. This measure gives us a relative error and our computations show that it is very small: the biggest measured relative error in our simulation step is 0.78%. To give more sense to this computation, we report it in figure 4. This plot has a double vertical axis. On the left axis we report all implemented government first choices with circles and all predicted replacement rates using the choice rule with crosses. These are ordered from the biggest deviation below the equilibrium replacement rate to the biggest one above. On the right axis we use a

[18]We only consider the first choices because the prediction errors are only important at that stage. Starting with the second choices, the predictions are so close to the actual choices that they are of a least concern.

bar plot to report the above error measure but relative to the biggest measured error of 0.78%, so that the biggest bar is scaled to 1. We note that the biggest errors appear when the initial deviations are closest to the equilibrium. In that case absolute errors are already extremely small and relative errors appear bigger only because the denominator in our indicator is also very small. Outside of this group of points, the errors are much smaller than the maximum measured error. We can conclude that overall, the choice rule is very accurate in predicting the actual behavior of the government.

Additionally, we also explore various alternative specifications for the laws of motion and choice rule. Our general findings are that our results are very robust to these changes and that we can somewhat reduce the complexity of our laws and rule without a significant impact on the results. A first change we consider is removing the law of motion of the aggregate asset from the model. A reason to do this would be, that contrary to, for example Krusell and Smith (1998), we do not need to predict future prices as we consider a partial equilibrium. Consequently, it could be that the average aggregate asset level has a less important role in the decision process of agents. Another reason would be the small coefficients relative to the aggregate asset we have in the law of unemployment, as detailed above. However, these reasons do not imply that following the aggregate financial asset might not be important for the government. We find that removing the law of motion of the aggregate asset does not modify sensibly enough the optimal time-consistent replacement rate. Only the law of motion of unemployment and the choice rule appear significant for this result: we can note that the unemployment rate has a direct impact on agents' utility through the tax rate whereas the aggregate financial asset only has an indirect effect in determining the evolution of the unemployment rate and the choice of the replacement rate. We have left the aggregate financial asset and the associated law of motion in the model because it helps us better explain the decomposition of the model dynamics: we can explicitly show that after a government decision there is an impact on aggregate asset level even though what really matters is each of the individual levels. Finally, we have also tried altering the law of motion of unemployment by introducing cross ($U_t x \rho_t$) and power ($U_t x U_t$ and $\rho_t x \rho_t$) terms. Again this does not change the optimal replacement rate significantly enough. However, we could think that it might change the dynamics around the equilibrium. For instance, if the current level of unemployment is already high, increasing the replacement rate will mechanically have a disincentive effect on a larger group of people and lead to a higher unemployment rate in the future. This will be captured by the cross term $U_t x \rho_t$ in the law of motion of

unemployment. Our alternative simulations show that the dynamics are indeed altered, but the quantitative effects are small and previous qualitative analyses are preserved.

3 The commitment case

In this section, we introduce –as a comparison to the fully time-consistent case above– a specification where the government can tie its hands about the implemented policy. Although it is not equivalent to the standard Ramsey policy[19], this specification provides good insights about the time consistency issue by being as close as possible to a full commitment case. We assume here that a once and for all shock to the replacement rate, immediately and indefinitely set to its new level, is implemented and that the future path of the economy is perfectly anticipated. Thus, given an initial state of the economy, we search for the replacement rate shock that maximize the utilitarian welfare criterion at time 0 in the economy. This is equivalent to performing a transition given an initial economy with a given policy to a new economy implementing another policy. However we consider a range of initial economies to account for the eventual dependence of the result to this initial state and for each initial state, we compute the optimal destination economy.

3.1 Model specification

For the most part household specifications are unchanged from the previous model and leads to the following program:

$$
\begin{aligned}
& V\left(a, \varepsilon, \rho, t\right) = \max_{c,s,a'} v(c,s) + \beta E V\left(a', \varepsilon', \rho, t+1\right) \\
& s.t. \\
& a' + c = (1+r)a + y(\varepsilon, \rho)(1-\tau) \\
& y(e, \rho) = w, \\
& y(u, \rho) = \rho w \\
& c \geq 0 \\
& a' \geq a_{min}
\end{aligned}
$$

[19]See for instance Ambler and Pelgrin (2010) for an exposition on how a standard Ramsey program is time-inconsistent.

Notice that because the time path of the economy is known, households display a simpler expectations behavior than in the time-consistent case.

We call ρ_{init} the replacement rate in the initial economy. Apart from running the unemployment insurance scheme, the role of the government in this case is limited to exogenously changing the replacement rate once at time zero. As before the budget of the unemployment insurance system is balanced at every date.

The optimal replacement rate $\rho^*(\rho_{init})$ is such that: (1) $\Psi_{\rho_{init}}(a, \varepsilon)$ is the only stationary distribution consistent with the constant replacement rate ρ_{init}; (2) Given the initial state $\Psi_{\rho_{init}}(a, \varepsilon)$, $\rho^*(\rho_{init})$ maximizes the $t = 0$ welfare criterion:

$$\rho^*(\rho_{init}) = \arg\max_{\rho} \left\{ \sum_{\varepsilon \in \{e,u\}} \int V_{\rho_{init}}(a, \varepsilon, \rho, 0)\, d\Psi_{\rho_{init}}(a, s) \right\}$$

where $V_{\rho_{init}}(a, \varepsilon, \rho, 0)$ is the individual intertemporal utility at the date of the once-and-for-all shock on ρ.

We can further define the deterministic transition as follows. The deterministic transition consists of initial conditions Ψ_0, the policy shock ρ, value functions $V(a, \varepsilon, \rho, t)$, the path of the tax rate $\tau_{t\geq 0}$, such that, at each date t:

1. Given $\tau_{t\geqslant 0}$ and ρ, $a'(a, \varepsilon, \rho, t)$ and $s(a, u, \rho, t)$ are the decision rules of the agent, solution to the previous program, and $V(a, \varepsilon, \rho, t)$ are the associated value functions,

2. Given Ψ_t, Ψ_{t+1} is generated by the computed decision rules $a'(a, \varepsilon, \rho, t)$ and $s(a, u, \rho, t)$,

3. The budget of the UI system is balanced at each date.

3.2 Results

In table 6, we report the results of the commitment case for various initial replacement rates. Our initial economies covers a wide range of replacement rates going from 20% to 70%. We find that whether the initial replacement rate is high or low, the optimal rate falls in a tight range just below 60%. But the smaller the initial UI rate and the smaller the final rate: quite intuitively when the initial rate is low, agent have better self-insurance and this mitigates the implementation of a higher replacement rate.

We compare the commitment results to the time-consistent case and observe that the optimal rates found here are either below or close to the time-consistent rate found for the longest choice periodicity of 25 years. These results are driven by the fact that agents in

the commitment case have different expectations than in the time-consistent case. First, the initial replacement rate change is not anticipated and comes as a surprise. Then, agents expect that the new rate will apply forever. Thus it is not surprising that the result should be close to the case where agents expect the replacement rate to be changed in a very distant future in the time-consistent case. The current government is able to fully benefits from the surprise redistributive effect the moment it sets the new replacement rate, then all subsequent governments are never expected to renege this policy, however inconsistent it might be.

Initial ρ (%)	20	30	40	50	60	70
Optimal ρ (%)	57.9	58.3	58.7	59.0	59.4	59.7
Final unemp. rate (%)	8.6	8.6	8.7	8.7	8.8	8.8

Table 6: Characteristics of the commitment model

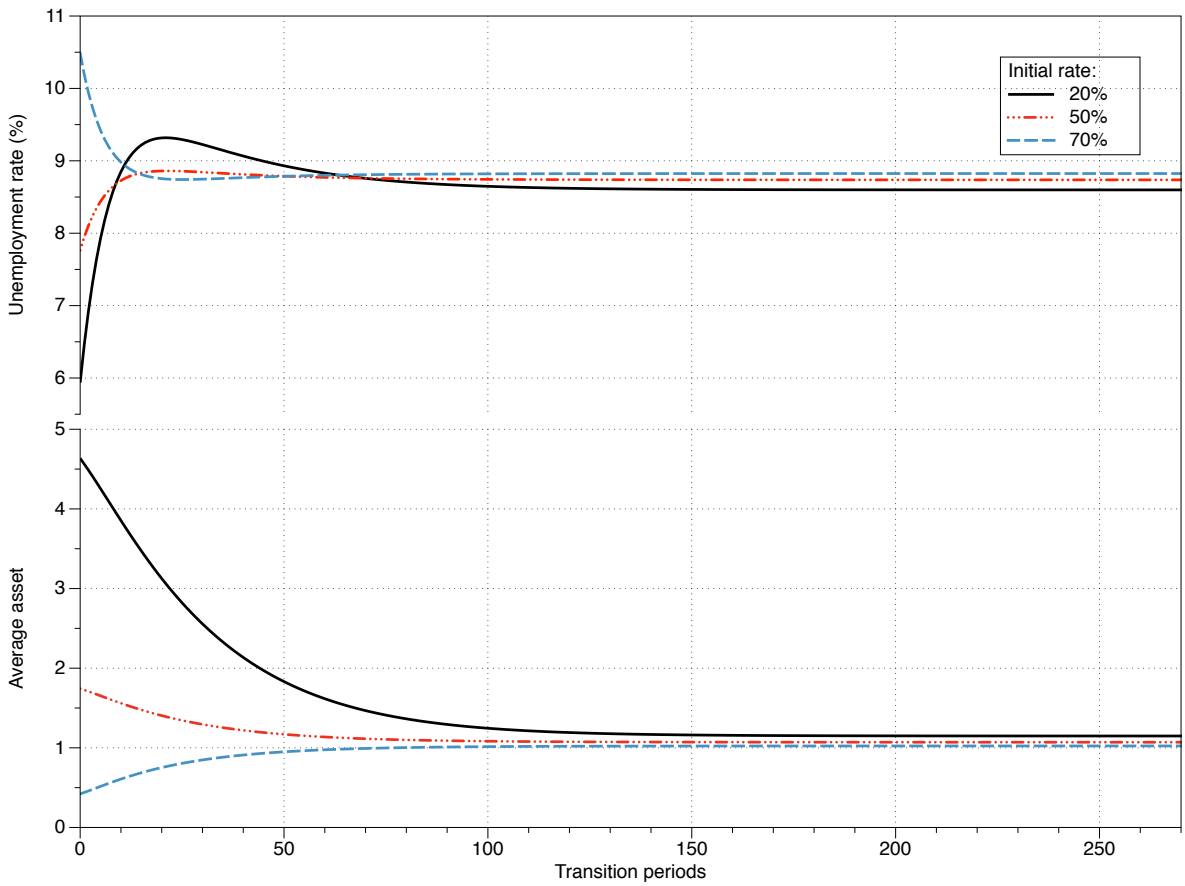

Figure 5: Transition path for average asset and unemployement rate.

In figure 5 we plot the transition path for average assets and unemployment rate for three of the initial economies we consider. By comparison with figure 2, we remark that the dynamics are qualitatively similar to those in the time-consistent case, when given a deviation away from equilibrium, the government has to wait a long time before making a first choice. The average assets converge monotonously and the unemployment rate adjustment is non monotonous. The unemployment dynamics can again be explained by the fact that it takes more time to accumulate assets than to adjust the unemployment for as long as the optimal self-insurance profile is not reached.

Overall and intuitively, the commitment case produces results close to the time-consistent case when the choice periodicity is quite long. However the time-consistent structure adds important trade-offs that have a significant impact on the optimal replacement rate. It is clear that the time-consistent setting adds a new layer of policy that favors higher replacement rates as the possibility to revise an implemented choice in the near future opens new perspectives.

4 Concluding remarks

The absence of a commitment device produces a number of interplays among the government, its future self and the economic agents with respect to the UI policy. The government has a temptation to immediately provide more public insurance as the disincentive effect only appear in the future and self-insurance can be relaxed. This temptation is stronger when the policy choice is decided more frequently. But we show that this result is dependent on the political decision system used. Furthermore the availability of an asset market matters in a very specific way in the determination of the optimal time-consistent replacement rate. These elements cannot be captured in a world where choices are not repeated and commitment is assumed.

Like other papers in this literature we have made simplifying assumptions to highlight the time-consistent behavior of the government in the context of repeated UI policy choices. For instance, we have ruled out any reputation building mechanism, assumed balanced budgets and only used a stylized labor market. We have also left out other important aspects of a UI policy such as the eligibility conditions for benefits or the degressivity of the replacement rates. We leave these issues for future research.

Acknowledgements

Appendix

A Approximate aggregation

We explain here how we circumvent the curse of dimensionality issue that appear when a distribution of agents is used in the model. We follow the approach in Krusell and Smith (1998) and approximate the distribution of agents by the mean value of asset holdings. The various rules (the laws of motion for the average financial wealth, the unemployment rate, and the choice rule), are all assumed to be linear. They can therefore be written as follows:

$$
\begin{aligned}
A_{t+1} &= \alpha_0^A + \alpha_1^A A_t + \alpha_2^A U_t + \alpha_3^A \rho_t \\
U_{t+1} &= \alpha_0^U + \alpha_1^U A_t + \alpha_2^U U_t + \alpha_3^U \rho_t \\
\rho_t &= \alpha_0^\rho + \alpha_1^\rho A_t + \alpha_2^\rho U_t
\end{aligned}
$$

The combination of the projection of the distribution of agents and the restriction to linear rules greatly reduces the dimensionality of the problem to be handled[20]. The recursive program of households can be rewritten as follows:

$$
\begin{aligned}
& V\left(a, \varepsilon, A, U, \rho\right) = \max_{c,s,a'} v(c,s) + \beta EV\left(a', \varepsilon', A', U', \rho'\right) \qquad (4) \\
& \text{s.t.} \\
& a' + c = (1+r)a + y(\varepsilon, \rho)(1 - \tau(\rho, U)) \\
& A' = \Gamma\left(A, U, \rho\right) = \alpha_0^A + \alpha_1^A A + \alpha_2^A U + \alpha_3^A \rho \\
& U' = \Theta\left(A, U, \rho\right) = \alpha_0^U + \alpha_1^U A + \alpha_2^U U + \alpha_3^U \rho \\
& \rho' = \Phi\left(A', U'\right) = \alpha_0^\rho + \alpha_1^\rho A' + \alpha_2^\rho U' \text{ with probability } \lambda \\
& \rho' = \rho \text{ with probability } (1-\lambda) \\
& y(e, \rho) = w \\
& y(u, \rho) = \rho w \\
& c \geq 0 \\
& a' \geq a_{min}
\end{aligned}
$$

[20] We also experiment with alternative rules to test the robustness of our specification.

with Θ the law of motion of the unemployment rate.

B Numerical implementation

In this section we characterize the algorithm used to find our reference results in the time-consistent economy using utilitarian welfare criterion. Other results and robustness tests use slightly different versions of this implementation when needed.

Among the state variable characterizing the agent's program, the distribution of agents Ψ is a mathematical object of infinite dimension. For numerical purposes, and following Krusell and Smith (1998), we assume that the distribution of agents can be approximated by its moments, and we restrict to its first order one, that is, the average financial asset holdings of agents A:

$$A_t = \int a\Psi_t(a)\,da$$

As the program 4 in Appendix A shows, there are 5 state variables, 2 of which are related to the household –individual financial wealth a_t, and employment ε_t– the remaining 3 -average financial wealth A_t, unemployment rate U_t, and current replacement rate ρ_t- being aggregate variables.

Apart from the employment status, which can take only 2 values (employed or unemployed), all other 4 variables are continuous ones. We resort to standard grid discretization techniques to approximate these variables. Uniform grids have been chosen. Regarding the aggregate variables, the range of possible values is not too wide, and the laws of motion are themselves linear. Moreover, as will become clearer below, once the equilibrium has been found, we can zoom in on these ranges, thus increasing the accuracy of our algorithm. As for the grid for individual asset holdings, uniform grids have also proved efficient and no accuracy issues were found after increasing the grid precision above our reference choice of precision.

In the function computing individual decision rules, we iterate over the Euler equation, interpolating linearly the future value for the marginal utility between two consecutive grid points. We need to iterate simultaneously on the marginal utility V' and the utility itself (the value function V), because the effort function itself depends on the intertemporal utility of the employed and unemployed agent.

The algorithm consists of a fixed point in the following 3 rules: (i) the choice rule that relates current average financial wealth A_t and unemployment U_t to the current choice of the government in terms of replacement rate ρ_t, (ii) the law of motion for the

unemployment rate U_{t+1} that relates this variable (future value) to A_t, U_t (current value for the unemployment rate) and ρ_t, and (iii) the law of motion for the average asset holdings A_{t+1} making it a function of A_t, U_t and ρ_t.

We only consider linear relations for these rules. The same argument as that pertaining to the definition of the grids applies here: once the equilibrium has been numerically found, we can consider small deviations around the equilibrium. In its neighborhood, linear relations are valid.

These rules are needed for agents to be able to forecast the future evolution of the economy. Once the expectations are taken into account, the government, whenever given the opportunity, will choose the level of the replacement rate ρ_t which maximizes the utilitarian criterion. Time-consistency is dealt with, since agents' intertemporal utility depends on the future government choices –apprehended through the choice rule Φ– which the current government takes as given. Note, however, that it is the function Φ which the government considers as exogenous: by changing the current replacement rate, the government can alter the path of the economy, and thus influence future choices dictated by function Φ.

The equilibrium, as could be expected, is stationary, so that these rules operate potentially, affecting agents expectations. We will end up with a single equilibrium, but it will depend on the 3 rules. To determine them, we then need to create some form of dynamics around the equilibrium. Indeed, these rules are forecasts of the dynamics in case the economy is hit by a replacement rate shock, if the public authorities decide to do so. Our strategy consists in:

1. Making an initial guess on values for the three rules,

2. Given these rules, computing the effort/saving decisions of households,

3. Given the household effort and savings rules and an initial guess on a converged stationary distribution, simulating the government choices of the replacement rate with probability λ,

4. Starting from the previous stationary distribution and the associated optimal replacement rate ρ^*, time zero ad hoc exogenous deviations of the replacement rate of various magnitudes are implemented, and time series with the government choosing a new policy (or retaining a previously chosen one) according to λ from each of these deviations are simulated. This creates the dynamics around the equilibrium. At each date, record A_t, U_t, ρ_t,

5. With the time series obtained for A_t, U_t, ρ_t, estimating the three rules, and updating them by a relaxation method, until the ex ante and ex post rules are close enough.

In practice, the three rules are not simultaneously updated. Rather, we proceed with nested loops, each of which is devoted to the convergence of one of the above three rules.

C Statistical model fit

We compute model fit statistics for the benchmark model in this section. We perform our computations for a baseline value of the choice parameter: the choice periodicity is 4 years. The model uses three linear rules: the laws of motion for the average financial wealth, the unemployment rate, and the choice rule.

$$
\begin{aligned}
A_{t+1} &= \alpha_0^A + \alpha_1^A A_t + \alpha_2^A U_t + \alpha_3^A \rho_t \\
U_{t+1} &= \alpha_0^U + \alpha_1^U A_t + \alpha_2^U U_t + \alpha_3^U \rho_t \\
\rho_t &= \alpha_0^\rho + \alpha_1^\rho A_t + \alpha_2^\rho U_t
\end{aligned}
$$

We compute the statistical fit for each rule in the converged model.

	R^2	$\hat{\sigma}$ (%)
Law of motion wealth	0.999999	0.001228
Law of motion unemp.	0.999989	0.000134
Choice rule	0.999728	0.001098

Table 7: Statistical properties of the linear prediction rules' fit

For each rule, both the R^2 and the estimate of the error standard deviation $\hat{\sigma}$ in percent are very good. Agents are able to predict the future level of average financial assets and unemployment with a very high accuracy. The same applies to the choice rule.

References

Acemoglu, D. and R. Shimer (2000). Productivity gains from unemployment insurance. *European Economic Review 44*(7), 1195–1224.

Aiyagari, S. R. (1994). Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics 109*(3), 659–684.

Ambler, S. and F. Pelgrin (2010). Time-consistent control in nonlinear models. *Journal of Economic Dynamics and Control 34*(10), 2215 – 2228.

Bewley, T. (1986). Stationary monetary equilibrium with a continuum of independently fluctuating consumers. *Contributions to Mathematical Economics in Honor of Gerard Debreu 79*.

Bruce, N. and M. Waldman (1991). Transfers in kind: Why they can be efficient and nonpaternalistic. *The American Economic Review*, 1345–1351.

Calvo, G. A. and M. Obstfeld (1988). Optimal time-consistent fiscal policy with finite lifetimes. *Econometrica: Journal of the Econometric Society*, 411–432.

Cohen, D. and P. Michel (1988). How should control theory be used to calculate a time-consistent government policy? *The Review of Economic Studies 55*(2), 263–274.

Costain, J. (1997). Unemployment Insurance with Endogenous Search Intensity and Precautionary Saving. Technical report, Department of Economics and Business, Universitat Pompeu Fabra.

Fischer, S. (1980). Dynamic inconsistency, cooperation and the benevolent dissembling government. *Journal of Economic Dynamics and Control 2*, 93–107.

Hansen, G. and A. İmrohoroğlu (1992). The role of unemployment insurance in an economy with liquidity constraints and moral hazard. *Journal of political economy 100*(1), 118–142.

Hillier, B. and J. M. Malcomson (1984). Dynamic inconsistency, rational expectations, and optimal government policy. *Econometrica: Journal of the Econometric Society*, 1437–1451.

Hopenhayn, H. and J. Nicolini (1997). Optimal Unemployment Insurance. *The Journal of Political Economy 105*(2), 412–438.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control 17*(5-6), 953 – 969.

Joseph, G. and T. Weitzenblum (2003). Optimal unemployment insurance: transitional dynamics vs. steady state. *Review of Economic Dynamics 6*(4), 869 – 884. Finance and the Macroeconomy.

Kankanamge, S. and T. Weitzenblum (2016). Time-inconsistency issues in designing unemployement insurance. *Mimeo*.

Klein, P., P. Krusell, and J. Rios-Rull (2008). Time-consistent public policy. *Review of Economic Studies 75*(3), 789–808.

Klein, P., V. Quadrini, and J. Rios-Rull (2005). Optimal time-consistent taxation with international mobility of capital. *BE Journals-Advances in Macroeconomics 5*.

Klein, P. and J.-V. Rios-Rull (2003). Time-consistent optimal fiscal policy. *International Economic Review 44*(4), 1217–1245.

Krusell, P. (2002). Time-consistent redistribution. *European Economic Review 46*(4-5), 755 – 769.

Krusell, P., V. Quadrini, and J.-V. Rios-Rull (1997, 1). Politico-economic equilibrium and economic growth. *Journal of Economic Dynamics and Control 21*(1), 243–272.

Krusell, P. and A. Smith (1998). Income and wealth heterogeneity in the macroeconomy. *The Journal of Political Economy 106*(5), 867–896.

Kydland, F. E. and E. C. Prescott (1977). Rules rather than discretion: The inconsistency of optimal plans. *The Journal of Political Economy 85*(3), 473–491.

Maskin, E. and J. Tirole (2001, 10). Markov perfect equilibrium: I. observable actions. *Journal of Economic Theory 100*(2), 191–219.

Pallage, S. and C. Zimmermann (2001). Voting on unemployment insurance. *International Economic Review*, 903–923.

Shavell, S. and L. Weiss (1979). The optimal payment of unemployment insurance benefits over time. *The Journal of Political Economy 87*(6), 1347–1362.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. *The Review of Economic Studies*, 165–180.

Wang, C. and S. Williamson (1996). Unemployment insurance with moral hazard in a dynamic economy. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 44, pp. 1–41. Elsevier.

Wang, C. and S. D. Williamson (2002). Moral hazard, optimal unemployment insurance, and experience rating. *Journal of Monetary Economics 49*(7), 1337 – 1371.

Young, E. R. (2004). Unemployment insurance and capital accumulation. *Journal of Monetary Economics 51*(8), 1683 – 1710.