Time-inconsistency issues in designing unemployment insurance*

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Abstract

In this article, we show, within a simple search model framework, that the choice of unemployment insurance implemented by a benevolent social planner depends on the periodicity of the public choice, because of time-inconsistency issues. We build an analytically tractable model where unemployment benefits are negatively affected by the periodicity of the public choice, and we show how a redistributive channel is responsible for this property. We then build and numerically solve a model of time-consistent public choices where the public authority, whenever choosing the optimal level of unemployment benefits, takes its future choices as given. Another channel is brought to light, explaining why here, too, benefits are negatively affected by the choice periodicity.

1 Introduction

In this article, we wish to raise the following question: do time consistency issues, related to the impossibility of the public insurance system to commit itself to future public choices, and to the repeated pattern of this public choice, affect the equilibrium unemployment benefits? Our analysis, built on both a theoretical approach, and a calibrated and simulated model, shows that an economy where the UI choice would be less frequently brought into the political debate, would be characterized by a lower level of unemployment benefits on average, and a higher degree of degressivity.

Time-consistency issues are well known to apply to monetary policy -the rules vs. discretion debate- and to optimal fiscal policy (see Kydland and Prescott (1977) both for the principle of time-inconsistency, and for its application to the monetary policy). Time-inconsistency arises because, when choosing today a certain value for a future, say, tax rate, this is likely to affect the behavior of agents over the whole time span separating the present from this future date. Once this date is reached, the public authority may

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find it worthwhile to reset the tax rate, because its impact on agents behavior may be different from then on. If the tax rate can be made time-dependent, it should be all the smaller as the agents' reaction to it is large (the tax base being very elastic), which is the case of future tax rates on capital, but not of current ones.

Redistributive schemes are also quite likely to give rise to time-inconsistency. For a given agent, this is obvious: today's choice over the current and future extent of redistribution depends on one's current state (whether one's current endowment is large or small), and in the future, there is no reason for one's individual endowment -let alone the global distribution of endowments- to remain unchanged. Public choices based, for example, on the median voter, depend, among others, on the distribution of endowments, on agents' mobility, and on the frequency of the public choice. Wright (1986) points out the redistributive role of UI, this time-inconsistency property, and the impact of the public choice periodicity on the equilibrium unemployment benefits replacement rate. Hassler and Mora (1999) study how the nature of labor market flows -the average durations of employment and unemployment- affects the equilibrium replacement rate chosen by the median agent, when choices on UI are operated at a given periodicity. Both articles ignore the potential disincentive impact of UI, as the probability transitions between employment and unemployment are exogenous to the workers. Hassler et al. (2005) analyze the impact of UI on workers mobility, when moving may help to find a new job, and the interactions between the majority voting procedure on UI benefits and the degree of workers mobility. As UI affects the decision of workers to move or not, unemployment benefits do affect the level of the unemployment rate. The impact of the periodicity of the public choice is however not analyzed, and if it were, it would be driven by redistributive channels and the preferences of the median voter over redistribution, similar to those of Wright (1986) and Hassler and Mora (1999).

The question of the optimal definition of unemployment insurance, regardless of these repeated public choice issues, but based on the optimal trade-off between insurance and incentives, has given rise to a very rich literature. It started with Shavell and Weiss (1979), where, within the principal-agent framework, the question of the UI time-profile which would minimize the cost of providing a given intertemporal utility to the unemployed, was raised. It was later extended by Hopenhayn and Nicolini (1997) who allow for a variable tax on re-employment. Both articles prove that, in the absence of individual savings, and when moral hazard takes the form of an unobservable search effort, the optimal time-profile of UI is degressive. More recently, much attention has been devoted to a more refined characterization of the optimal time-profile for UI when agents can save and for a specific type of utility function (CARA utility functions in Werning (2002)). In particular, the optimality of a degressive time-profile, first put forward by Shavell and Weiss (1979), does not carry over to an environment where hidden savings are taken into account (Werning (2002) and Shimer and Werning (2008)). The different implications of considering either a pure search model with wage draws, or a model where the wage level is unique, and agents choose their search intensity, are also thoroughly explored (again, Werning (2002) and Shimer and Werning (2008)).

Parallel to these studies based on the principal-agent framework, quantitative macro models have also been built, initially to allow for private financial savings. Hansen and İmrohoroğlu (1992) first introduced unemployment risk in a model where agents can save, but do not have access to an insurance market against income risk. Their article essentially focused on the interaction between moral hazard -the ability, for unemployed, to turn down job offers without being monitored- and the saving behavior, working as a self-insurance device. A large number of articles followed (Pallage and Zimmermann (2001), Acemoglu and Shimer (2000), Costain (1997)), among which Wang and Williamson (2002) numerically computed the optimal time-profile of UI with private savings, and showed that this profile was non-monotonic. Although the two different approaches -principal-agent vs. simulated macro models- differ in the methodology used, the questions regarding the impact of the definition of UI on the degree of insurance and the disincentive effects simultaneously generated, are identical, as well as the underlining intuitions.

This article reconsiders the optimal trade-off between insurance and incentives, through the time-inconsistency issues which arise when the public choice of UI is operated by a benevolent social planner. It is therefore located at the crossroads of these two strands of the literature. While our approach shares with Wright (1986) and Hassler and Mora (1999) the public choice dimension, it does, contrary to these two papers, take the disincentive impact of UI into account. In particular, the utilitarian criterion as the public choice function gives rise to time-inconsistency, while its application was not relevant in the two papers, which were grounded on median agent voting procedures. We wish to shed light on the reasons why the trade-off between insurance and incentives depends on the periodicity of the public choice. To this end, we consider a search model, where unemployed freely choose their search intensity. Apart from the public choice, the framework is standard in the theoretical literature on the optimality of UI.

In the absence of a commitment technology, the current public choice, known to apply for a given time span, is likely to depend on the choice periodicity. Indeed, in the case of unemployment insurance, the periodicity will affect both the disincentive effect of a given increase in the replacement rate, and the obtained insurance -and, in some cases, pure redistributive- gains. In particular, when an unexpected new increase in the replacement rate immediately applies, it does, at least for the current period, give rise to pure redistribution, with no disincentive impact, as only future unemployment benefits affect the current search effort of the unemployed. However short-lived this gain may be, it will be part of the global trade-off faced by the public insurance agency; in particular, this gain, experienced only at the current date, does not depend on the public choice periodicity. As for the disincentive effect of UI, it will, quite intuitively, be all the higher, as the public choice periodicity is greater. Consequently, the periodicity of public choices will affect the relative weight of the pure redistributive mechanism: the shorter the periodicity, the higher the temptation to benefit from current redistributive gains, and the higher the equilibrium replacement rate. We develop a simple analytical model to highlight this trade-off, and the impact of the periodicity of the UI choice. For the model to be fully tractable, we however do not consider explicitly repeated choices, but resort to a specific equilibrium definition which captures, hopefully, the salient features of repeated choices, without handling them upfront.

We also shed light on another, less intuitive, channel, through which the public choice periodicity affects the equilibrium replacement rate. To this end, we turn to a fully time-consistent approach. The equilibrium definition follows that of papers based on tractable (either analytically, or numerically) versions of recursive politico-economic equilibria, which address the issue of time-consistency of public choices by considering that no commitment is possible for the public authority. Whenever a current choice is decided upon, expectations regarding future choices are made, and at the equilibrium, these expectations turn out to be correct (see Krusell et al. (1997) for a general presentation)[1].

The model is calibrated and simulated, for various public choice periodicities. The above-mentioned redistributive channel is neutralized, by considering that when a new choice for the replacement rate is made, it will apply only from the next period on. The simulations show that the equilibrium replacement rate is, here also, a decreasing function of the public choice periodicity. We then numerically show that another mechanism is at work: the disincentive impact of an announcement of a future increase in the replacement rate is all the greater, as the increase is expected to take place in a more distant future. Finally, we illustrate the impact of the public choice periodicity on the equilibrium UI time-profile -considering a basic time-profile consisting of two different replacement rates (one for short term unemployed, and one for long term ones). The equilibrium time-profile of UI is more degressive, when the choice on UI is repeated less often. The average level of unemployment benefits, however, is modestly affected by the frequency of the public choice.

The rest of the article is structured as follows. Section 2 presents and solves the analytical model. Section 3 presents the fully time-consistent model, its calibration, and the various simulations which are implemented. Section 4 briefly concludes.

2 A simple analytical model

2.1 Households preferences

The economy consists of a continuum of households of measure one, who either work, or are unemployed, at each date. Households preferences are represented by a time-separable additive expected utility, which writes:

$$E_0\left(\sum_{t=0}^{\infty} u(c_t) - g(\pi_{t+1})\right)$$

where c_t and π_{t+1} respectively denote date t consumption and the probability to be hired at the beginning of date $t+1$. The instantaneous utility $u(c)$ is such that $u'(c) > 0, u''(c) < 0$. By assumption, savings are precluded. Consequently, households consume their entire income at every period. At any given date t, all agents –unemployed and employed– choose their probability of being hired at the next period, π_{t+1}. This assumption is equivalent to stating that jobs last for a single time period, and that employed and unemployed agents search with the same efficiency. It also implies that tomorrow's unemployment rate $U_{t+1} = 1 - \pi_{t+1}$.

The probability of being hired entails a disutility $g(\pi_{t+1})$ with the function g such that $g'(0) = 0, g'(\pi) > 0$ and $g''(\pi) > 0$ for $\pi > 0$, and

$$\lim_{\pi \to \overline{\pi}} g(\pi) = +\infty$$

[1]This formulation of the equilibrium, requiring the consideration of subtle dynamics, has, among others, been applied to redistributive questions (Krusell (2002)).

with $\overline{\pi}$ a constant parameter strictly below 1. This assumption amounts to impose that, no matter what the current search effort is, the probability of being unemployed tomorrow is bounded below by a strictly positive value.

When employed, households earn an exogenous wage w and pay taxes τ_t. Unemployed households receive benefits $b = \rho_t w_t$, independent of the duration of their current unemployment spell.

The expected intertemporal utility of both types of agents can be formulated using the standard Bellman equations. The current state of an agent consist of her current employment status (e for employed, u for unemployed), and the current state of the economy. The latter comprises the current unemployment rate U_t and the current replacement rate ρ_t. We therefore have:

$$\begin{aligned} V_e^t = V_e(\rho_t, U_t) = \max_s \{ & u(w(1-\tau(\rho_t, U_t))) - g(\pi_{t+1}) \\ & + \beta[\pi_{t+1}(E_t V_e(\tilde{\rho}, U_{t+1})) + (1-\pi_{t+1})(E_t V_u(\tilde{\rho}, U_{t+1}))]\} \end{aligned}$$

$$\begin{aligned} V_u^t = V_u(\rho_t, U_t) = \max_s \{ & u(\rho_t w) - g(\pi_{t+1}) \\ & + \beta[\pi_{t+1}(E_t V_e(\tilde{\rho}, U_{t+1})) + (1-\pi_{t+1})(E_t V_u(\tilde{\rho}, U_{t+1}))]\} \end{aligned}$$

where tomorrow's replacement rate $\tilde{\rho}$ is not known with certainty, as of date t (to be explained below). The chosen unemployment exit-rate (or, equivalently, the chosen search effort) satisfies the following first-order condition[2]:

$$-g'(\pi_{t+1}) + \beta E_t \left(V_e(\tilde{\rho}, U_{t+1}) - V_u(\tilde{\rho}, U_{t+1}) \right) = 0$$

which simplifies to:

$$g'(\pi_{t+1}) = \beta E_t \left((u(w(1-\tau(\tilde{\rho}, 1-\pi_{t+1}))) - u(\tilde{\rho} w) \right))$$

where U_{t+1} has been expressed as a function of the current effort π_{t+1}. From this expression, it is straightforward to notice that the search effort only depends on the process governing $\tilde{\rho}$. In particular, tomorrow's unemployment rate is independent of today's.

2.2 The unemployment insurance

Unemployment insurance is compulsory, and financed by contributions levied on the gross wage w. The budget of the public insurance system is assumed to be balanced at every date. That is:

$$w\tau_t(1-U_t) = \rho_t w U_t$$

This leads to the following expression for the tax rate

$$\tau_t = \tau(\rho_t, U_t) = \frac{\rho_t U_t}{1-U_t}$$

τ being a function of two variables, we further define its two partial derivatives as:

[2]The second order condition is verified, given the assumption on $g(\pi_{t+1})$.

$$\frac{\partial\tau(\rho,U)}{\partial\rho} = \tau_1'(\rho,U) = \frac{U}{1-U}$$
$$\frac{\partial\tau(\rho,U)}{\partial U} = \tau_2'(\rho,U) = \frac{\rho}{(1-U)^2}$$

2.3 The choice over unemployment benefits

The replacement rate ρ_t is chosen by the public insurance agency at the beginning of the initial date, $t = 0$. This replacement rate applies with certainty at the current date, and with probability λ, it will be kept next period, and so on, from a time period to the next. The replacement rate applies to all unemployed at $t = 0$, even those who were already unemployed at the previous period: this new policy is therefore retroactive[3].

The agency chooses the replacement rate, so as to maximize the utilitarian criterion, evaluated at the current date. However, this choice will apply only for a limited time span, and the insurance agency expects the replacement rate to be thereafter reset to a constant level, $\overline{\rho}$. Consequently, the public choices are not repeated in this setting, and are not time-consistent. However, as will become clearer, the temptation to deviate toward higher unemployment benefits, and its dependence on the frequency of the political choice, will be present, and these channels are also at work in the full time-consistent approach.

From here on, time subscripts for the replacement rate will be dropped: at any date, either the replacement rate is still that chosen at date $t = 0$, denoted ρ', or it has reverted back to its exogenous long-run level, $\overline{\rho}$. Given the time-structure for the public choice, the first-order condition on the search effort at date $t = 0$ is:

$$g'(\pi_1) = \beta E_0 \left[\lambda(u(w(1-\tau(\rho', U_1))) - u(\rho' w)) + (1-\lambda)(u(w(1-\tau(\overline{\rho}, U_1))) - u(\overline{\rho} w))\right] \tag{1}$$

Apart from the exogenous parameters, the search effort depends on both $\overline{\rho}$ and ρ'. The definition of the equilibrium (see below) requires to consider marginal deviations of ρ', for a given $\overline{\rho}$. These two replacement rates will therefore not be treated symmetrically. Next period's unemployment rate can be written as a function of ρ', parameterized by $\overline{\rho}$. To simplify the notations, the reference to $\overline{\rho}$ will be dropped in the following calculations:

$$U_1 = \hat{u}_{\overline{\rho}}(\rho') = \hat{u}(\rho')$$

The value functions V_e and V_u can then be expressed more explicitly:

$$\begin{aligned} V_e(\rho', U_0) = \max_{\pi_1}\{&u(w(1-\tau(\rho', U_0))) - g(\pi_1) + \beta[\pi_1(\lambda V_e(\rho', \hat{u}(\rho')) + (1-\lambda)V_e(\overline{\rho}, \hat{u}(\rho'))) \\ &+ (1-\pi_1)(\lambda V_u(\rho', \hat{u}(\rho')) + (1-\lambda)V_u(\overline{\rho}, \hat{u}(\rho')))]\} \end{aligned}$$

[3]In the real world, unemployed usually sign a contract with the public administration in charge of the UI program at the beginning of their unemployment spell. This assumption is therefore counterfactual, but keeping track of different types of unemployed -those with replacement rates chosen in the past, and the newly unemployed for which the current choice on the replacement rate would immediately apply- would be very burdensome. Besides, only deviations from the original contract which are not favorable to the unemployed would be opposed by them. As will become clear later on, frequent public choices, on the opposite, lead to temptations to deviate toward higher benefits, which unemployed would gladly accept.

$$V_u(\rho', U_0) = \max_{\pi_1}\{u(\rho' w) - g(\pi_1) + \beta[\pi_1(\lambda V_e(\rho', \hat{u}(\rho')) + (1-\lambda)V_e(\overline{\rho}, \hat{u}(\rho'))) + (1-\pi_1)(\lambda V_u(\rho', \hat{u}(\rho')) + (1-\lambda)V_u(\overline{\rho}, \hat{u}(\rho')))]\}$$

When the insurance agency sets the replacement rate, the current state of the economy is obviously a crucial determinant. Here, the state of the economy is entirely described by the current unemployment rate, U_0. The program of the public insurance agency writes:

$$\max_{\rho'} W_0 = (1-U_0)V_e(\rho', U_0) + U_0 V_u(\rho', U_0)$$

The first order condition for the public insurance agency program is:

$$(1-U_0)\frac{dV_e(\rho', U_0)}{d\rho'} + U_0\frac{dV_u(\rho', U_0)}{d\rho'} = 0 \tag{2}$$

2.4 The equilibrium

As mentioned earlier, the political choice occurs only at date $t = 0$, and everyone knows that the replacement rate will be indefinitely set at $\overline{\rho}$ once the one-time choice no longer applies. Given initial conditions -consisting of the initial unemployment rate U_0-,and given $\overline{\rho}$, the insurance agency will choose to deviate and opt for ρ' which is the solution of the previous equation.

We define the politico-economic equilibrium as the vector (U_0, ρ') such that:

- the initial unemployment rate U_0 is the only one compatible with $\rho' = \overline{\rho}$, and the optimality condition (1): $U_0 = \hat{u}(\overline{\rho})$,
- given U_0, and for a future constant replacement rate $\overline{\rho}$, the agency finds it optimal not to deviate from its previously chosen replacement rate, that is, ρ' solves equation (2) with $\rho' = \overline{\rho}$.

The equilibrium has the desired property that it is stable with respect to the political process: if $\overline{\rho}$ has been applied in the past without any deviation from it, and if the public insurance agency has the possibility to deviate at date $t = 0$, it will choose not to. What lacks in this simple setting, as compared to a fully sketched time-consistent political choice, is (i) the fact that we do not know how the economy has reached its initial state and (ii) the repeated public choice. Indeed, the public choice only occurs at date $t = 0$, and is not expected to take place in the future. However simple this economy may be, it will be suited for (i) recovering the Baily-Chetty (Baily (1978) and Chetty (2006)) optimality condition on the replacement rate and (ii) underlining the impact of the periodicity of the public choice (the value of λ) on the short-run temptation to deviate toward higher replacement rates and, in turn, on the equilibrium replacement rate.

Our goal here consists in analytically characterizing the impact of the expected duration of the change in the replacement rate on the equilibrium replacement rate.

2.5 Recovering the Baily-Chetty optimality condition

We can state the following proposition:

Proposition 2.1. *At the equilibrium, the necessary first order condition for the maximization of the utilitarian criterion is:*

$$\frac{u'(\rho' w) - u'\left(w(1-\tau(\rho', U_0))\right)}{u'\left(w(1-\tau(\rho', U_0))\right)} = \frac{\beta\rho'\hat{u}'(\rho')}{U_0(1-U_0)} \tag{3}$$

Proof. See Appendix B.1. □

This condition is the Baily-Chetty optimality condition, adapted to this framework. The left-hand side is the relative gap in terms of marginal utility, between an employed and an unemployed. The lower it is, the better is the public insurance against the unemployment risk. The right-hand side indicates how costly, in terms of disincentive, the insurance is. At the optimum, gains in terms of insurance, and costs in terms of disincentive, are equated. The right-hand side can be rewritten in terms of the elasticity of unemployment with respect to benefits $\varepsilon_{U/\rho'}$:

$$\frac{\beta\rho'\hat{u}'(\rho')}{U_0(1-U_0)} = \frac{\beta\varepsilon_{U/\rho'}}{1-U_0}$$

2.6 Closing the model

The previous optimality condition is based on the function $\hat{u}(\rho')$, which itself derives from the behavior of the household. To close the model, we further characterize this function. We differentiate the optimality condition (1), valid for a given ρ', with respect to ρ':

$$\begin{aligned} g''(\pi_1)\frac{d\pi_1}{d\rho'} = \beta \Bigg[\lambda \left(\frac{\partial V_e}{\partial \rho'}(\rho', \hat{u}(\rho')) - \frac{\partial V_u}{\partial \rho'}(\rho', \hat{u}(\rho')) \right) \\ +(1-\lambda) \left(\frac{\partial V_e}{\partial \rho'}(\overline{\rho}, \hat{u}(\rho')) - \frac{\partial V_u}{\partial \rho'}(\overline{\rho}, \hat{u}(\rho')) \right) \Bigg] \end{aligned} \tag{4}$$

The expression of the disincentive effect, $\hat{u}'(\rho')$, is provided in the following proposition:

Proposition 2.2. *The impact on unemployment of a marginal deviation in the replacement rate (the disincentive effect), $\hat{u}'(\rho')$, is:*

$$\hat{u}'(\rho') = \frac{\beta\lambda w \left(u'(\rho' w) + \frac{\hat{u}(\rho')}{1-\hat{u}(\rho')} u'(w(1-\tau(\rho', \hat{u}(\rho')))) \right)}{g''(1-\hat{u}(\rho')) - \frac{\beta\rho' w}{(1-\hat{u}(\rho'))^2} u'(w(1-\tau(\rho', \hat{u}(\rho'))))} \tag{5}$$

with $g''(1-\hat{u}(\rho')) - \frac{\beta\rho' w}{(1-\hat{u}(\rho'))^2} u'(w(1-\tau(\rho', \hat{u}(\rho')))) > 0$ *for* $\rho' = \overline{\rho} < \rho^{max}$, ρ^{max} *being the highest replacement rate compatible with an unemployment rate* $\hat{u}(\rho') < 100\%$.

Proof. The proof consists of two different parts. The first part derives the above-mentioned expression and is detailed in Appendix B.2. The second part proves that there exists a higher bound for the replacement rate $\overline{\rho}$ below which agents exert a positive search effort (otherwise, nobody would search and no production would take place in this economy). It is detailed in Appendix A, as one of the properties of the stationary model. □

The impact of the persistence of the deviation in the replacement rate, λ, is unambiguous: $\hat{u}'(\rho')$ is higher, the higher is λ. In other words, a given marginal deviation in ρ' will have a stronger impact on the future unemployment rate, when the deviation is expected to last longer. Indeed, this marginal deviation will affect the search effort insofar as it reduces the gain in utility from being hired. Therefore, the marginal increase in ρ' will have a greater impact on the marginal reduction in the gain from being hired, when the deviation is more persistent. This mechanism, as can be seen, materializes as a proportionate effect of λ on $\hat{u}'(\rho')$.

Finally, the Baily-Chetty condition can be rewritten as follows, after imposing $U_0 = \hat{u}(\rho')$ at the equilibrium:

$$\frac{u'(\rho' w) - u'\left(w(1-\tau(\rho', U_0))\right)}{u'\left(w(1-\tau(\rho', U_0))\right)} = \frac{\rho'\beta^2\lambda w\left(\frac{u'(\rho' w)}{U_0(1-U_0)} + \frac{u'(w(1-\tau(\rho',U_0)))}{(1-U_0)^2}\right)}{g''(1-U_0) - \frac{\beta\rho' w}{(1-U_0)^2}u'(w(1-\tau(\rho',U_0)))} \tag{6}$$

Each of the two parts of this relation depends on both the replacement rate ρ' and the initial unemployment rate U_0. At the equilibrium, U_0 is the only unemployment rate compatible with a replacement rate of $\overline{\rho}$, which is not subject to deviation. To state it differently, at the date $t = -1$, all households expect the future replacement rate to be equal to $\overline{\rho}$, and their search effort results in unemployment at date $t = 0$ being equal to U_0. Thus, U_0 is itself a function of $\overline{\rho}$ [4]: $U_0 = \Upsilon(\overline{\rho})$. As shown in Appendix A, there exists only one stable equilibrium, for a given replacement rate $\overline{\rho}$ expected with certainty. Quite intuitively, the unemployment rate is an increasing function of the replacement rate: $\frac{d\Upsilon(\overline{\rho})}{d\overline{\rho}} > 0$. Finally, as we are only interested in finding the equilibrium, we have already imposed that $\overline{\rho} = \rho'$. This implies that the two sides depend only on the replacement rate ρ'.

The left-hand side is unambiguously decreasing in ρ' (and trivially continuous): as ρ' increases, the marginal utility of an unemployed is reduced, the unemployment $U_0 = \Upsilon(\overline{\rho}) = \Upsilon(\rho')$ rises, so the after-tax wage goes down, and the marginal utility of the employed increases. Also, the left-hand side becomes arbitrarily high, as ρ' gets closer to 0. Finally, recall that there exists a higher bound on the replacement rate, above which no production takes place. Figure 1 plots a possible case.

In the general case, the right-hand side is not monotonous. It is therefore not possible to prove the uniqueness of the equilibrium. We can, however, prove that at least one stable equilibrium exists, and, that, when a certain condition on the function $g(\pi)$ is met, it is unique.

Proposition 2.3. *For $\lambda > 0$, there exists at least one interior value $\rho' \in]0; \rho^{max}[$ solution of (6).*

[4] The function $\Upsilon(\rho)$ defines the relation between unemployment benefits and the unemployment rate in a stationary environment deprived of temporary deviations. In particular, one has: $\Upsilon(\rho) = \hat{u}_\rho(\rho)$.

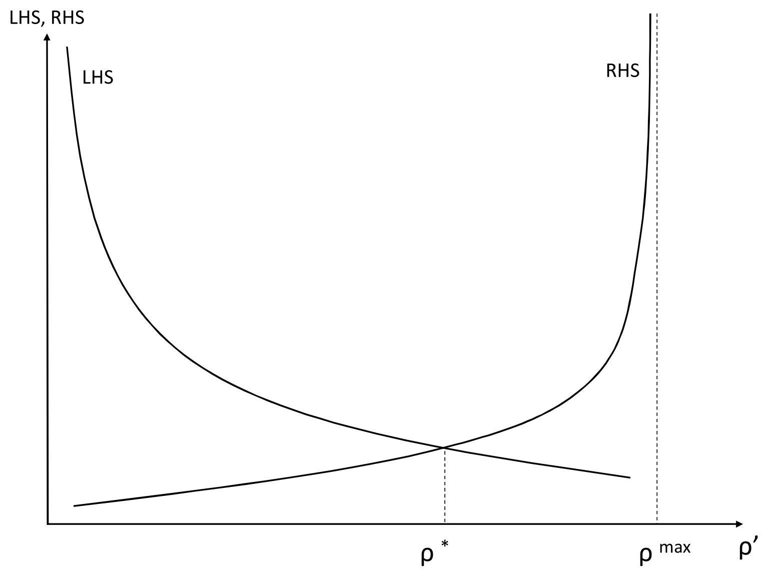

Figure 1: Equilibrium characterization: a case study.

Proof. See Appendix B.3. ☐

Proposition 2.4. *A sufficient condition for the existence and the uniqueness of the equilibrium, regardless of the value of* $\lambda > 0$, *is* $(1-\pi)g'''(\pi) > g''(\pi), \quad \forall \pi, 0 \leqslant \pi \leqslant \overline{\pi}$

Proof. See Appendix B.4. ☐

Proposition 2.5. *When the solution to equation (6) is unique, a marginal increase in* λ *results in a reduction of the replacement rate.*

Proof. At the left of ρ^*, the left-hand side is greater than the right-hand side, and the opposite applies at the right of ρ^*. An increase in λ leaves the left-hand side unchanged, while it pushes upward the right-hand side. It follows that the intersection moves toward lower levels of ρ. ☐

2.7 Interpreting the equilibrium condition

Several mechanisms are simultaneously at work in this setting: a marginal increase in benefits generates gains in terms of insurance, as well as costs in terms of disincentive effect on the unemployment rate. In addition, the choice, operated at $t = 0$, has an immediate and pure redistributive impact. Redistribution and insurance differ insofar as the former takes place *ex post*, reallocating wealth from high to low income agents without having been anticipated, while the latter takes place *ex ante*, operating the same reallocation, but being properly anticipated. Here, an unexpected increase in the date $t = 0$ unemployment benefit is redistributive, but it exerts no disincentive: today's unemployment rate is predetermined. The disincentive bears on the impact of future benefits on the future unemployment rate.

To isolate this channel, we can consider, as a benchmark, the case where the choice on the replacement rate, undertaken at $t = 0$, will apply only from date $t = 1$ on. In this manner, the pure redistributive impact is offset. From $t = 2$ on, the shock ceases to apply with probability λ. We will refer to this version of the model as the no surprise model. When needed, the full model where the deviation takes place at date $t = 0$ will be denoted the baseline.

In the no surprise case, one can show (see Appendix C) that, the optimal choice of the replacement rate satisfies another version of the Baily-Chetty condition[5]:

$$\frac{u'(\rho' w) - u'\left(w(1-\tau(\rho', U_0))\right)}{u'\left(w(1-\tau(\rho', U_0))\right)} = \frac{\rho' \hat{u}'_{\lambda=1}(\rho')}{U_0(1-U_0)} \tag{7}$$

This optimality condition is independent of λ, although λ affects the expected duration of the policy shock[6]. Precisely, one would obtain exactly the same condition, for a deviation, chosen at date $t = 0$, and applying with certainty only at date $t = 1$ (or until any terminal date t_{final} known with certainty). This can be regarded as the optimal trade-off between insurance and efficiency, absent any redistributive issue. Qualitatively, and with the help of the previous graphical analysis, the right-hand side of equation (7), representing the increasing curve, is higher than in the baseline case[7], so the equilibrium corresponds to a lower replacement rate.

Let us now analyze the impact of λ in the baseline case. The equilibrium replacement rate balances the current redistributive gains with the impact this marginal deviation will have in the future. The global future impact consists of the discounted sum of the impacts of a marginal upward deviation of the replacement rate on the expected utility, at each of the future dates at which it will take place. Conditional on the deviation being still carried on at date $T-1$, the future marginal effect FME_T^{λ} at date $T-1$ writes:

$$\begin{aligned} FME_T^{\lambda}(\rho') = \frac{d}{d\rho'}&\left[-g(\pi_T) + \beta\left(\lambda\left(\pi_T u(w(1-\tau(\rho',\hat{u}(\rho')))) + (1-\pi_T)u(\rho' w)\right)\right.\right. \\ &\left.\left. + (1-\lambda)\left(\pi_T u(w(1-\tau(\overline{\rho},\hat{u}(\rho')))) + (1-\pi_T)u(\overline{\rho} w)\right)\right)\right] \\ = \beta\lambda &\left[\hat{u}(\rho')w(u'(\rho' w) - u'(w(1-\tau(\rho',\hat{u}(\rho'))))) - \frac{\rho' w \hat{u}'(\rho')}{\lambda(1-\hat{u}(\rho'))}u'(w(1-\tau(\rho',\hat{u}(\rho'))))\right] \\ = \beta\lambda\hat{u}(\rho')u'(w(1-\tau(\rho',\hat{u}(\rho'))))w &\left[\frac{u'(\rho' w) - u'(w(1-\tau(\rho',\hat{u}(\rho'))))}{u'(w(1-\tau(\rho',\hat{u}(\rho'))))} - \frac{\rho' \hat{u}'(\rho')}{\lambda\hat{u}(\rho')(1-\hat{u}(\rho'))}\right] \end{aligned} \tag{8}$$

Consider the second to last expression, where the channels appear more distinctly. The first term inside the brackets corresponds to the insurance gains of a marginal increase in the replacement rate at any date -not only the next one-, conditional on being

[5]It is easy to show that the condition in proposition (2.4) is also a sufficient condition for existence and uniqueness of the equilibirum in the no surprise case.

[6]In this setup, once a marginal deviation in ρ' has been decided at $t = 0$, it will apply with certainty in $t = 1$. Therefore, the "true" marginal effect of a deviation in ρ', $\hat{u}'(\rho')$, corresponds to equation (5) with $\lambda = 1$.

[7]Precisely, the right-hand side of (7) is equal to the right-hand side of equation (3) divided by $\beta\lambda$, so it is clearly larger.

implemented. In this stylized setup where employed and unemployed face the same employment prospects, the insurance gains exactly correspond to the impact of a marginal increase in ρ' on the utilitarian criterion. The disincentive costs correspond to the second term inside the brackets. The benefit increase raises the unemployment rate, which translates into a tax increase paid exclusively by employed agents. We can also notice that:

$$FME_T^{\lambda}(\rho') = \lambda FME_T(\rho')$$

where $FME_T(\rho')$ is the marginal effect of an upward deviation of the replacement rate evaluated at date $T-1$, when this deviation will be for sure ($\lambda = 1$) maintained at date T. These marginal effects are all equal: $FME_T(\rho') = FME_{T'}(\rho') = FME(\rho'), T' \neq T$. The global future marginal effect, seen from date $t = 0$, is simply

$$\sum_{t=1}^{\infty}(\beta\lambda)^{t-1}FME_T^{\lambda}(\rho') = \sum_{t=1}^{\infty}(\beta\lambda)^{t-1}\lambda FME(\rho') = \frac{\lambda}{1-\beta\lambda}FME(\rho')$$

In the baseline model, a benefit increase also gives rise to pure redistributive gains $RG_0(\rho')$ at date $t = 0$, which write[8]:

$$RG_0(\rho') = \hat{u}(\rho')w(u'(\rho' w) - u'(w(1-\tau(\rho', \hat{u}(\rho')))))$$

At the optimal public choice, the sum of all marginal effects is equal to zero:

$$RG_0(\rho') + \frac{\lambda}{1-\beta\lambda}FME(\rho') = 0$$

As instantaneous redistributive gains always occur ($RG_0(\rho') > 0$), the public authority will decide to take advantage of them, up to the point where the deviation generates future marginal costs exactly balancing the instantaneous marginal gains. Formally, in expression (8), $\hat{u}(\rho') = U_0$, and the bracketed term is negative (since condition (3) is verified), which implies that $FME(\rho') < 0$.

The expected duration of the public choice affects the equilibrium in the baseline case, because at the optimal public choice, the marginal effect at any future date is negative ($FME(\rho') < 0$), so the total marginal effect will depend on how long these costs will be suffered. If the public choice is expected to last longer, the total future costs of any replacement rate increase will be larger, so the public authority will opt for a lower replacement rate. In the end, this is driven by the fact that the pure redistributive gains occur for exactly one period, while the future costs will take place for a variable length of time.

At the limit, when $\lambda = 0$, a pure redistribution is operated at $t = 0$, with no impact whatsoever on the search effort of agents, as, from $t = 1$ on, the replacement rate is not affected by this deviation. From condition (3), noting that $\hat{u}'(\rho') = 0$ in this case, it is clear that the after-tax income will be perfectly equated between the employed and the

[8]One can notice that formally, the insurance effect and the redistributive effect are identical. As already mentionned, what distinguishes them, is only the fact that insurance takes place *ex ante*, while redistribution, operated as a surprise, takes place *ex post* at date $t = 0$, without have been anticipated at the previous date.

unemployed. This, in turn, implies that agents have no incentive to search, so that in this economy, all agents remain unemployed.

In the no surprise case, in the absence of the pure initial redistribution, only the trade-off between insurance and disincentive effects takes place. Formally, the total effect of a marginal replacement rate increase is $\frac{\lambda}{1-\beta\lambda}FME(\rho')$. The optimality condition imposes:

$$\frac{\lambda}{1-\beta\lambda}FME(\rho') = 0 \Leftrightarrow FME(\rho') = 0$$

This optimality condition is the same, whether it is imposed over the whole expected time span, or for a single model period.

Figure 2 provides a numerical illustration of the impact of λ on the equilibrium replacement rate in the baseline case where the deviation occurs at date $t = 0$. We adopt the following specifications:

$$u(c) = \frac{C^{1-\gamma}}{1-\gamma}, \qquad\qquad g(\pi) = A\left(\frac{1}{\xi}ln\left(\frac{\overline{\pi}}{\overline{\pi}-\pi}\right)\right)^{\psi}$$

with $\gamma = 2$. This specification can be better understood by introducing an intermediary variable: the search effort s. It indeed derives from a disutility of effort $g(s) = As^{\psi}$ and an unemployment exit rate $\pi(s) = \overline{\pi}(1 - e^{-\xi s})$. We impose $A = 0.01, \psi = 3.0, \xi = 1.5, \overline{\pi} = 0.97, \beta = 0.98$. The figure below plots the curves for the left hand-side and the right-hand side, for various values for λ. In this case, the right-hand side is monotonously increasing, and the equilibrium is unique. The longer the initial deviation is on average (that is, the higher is λ), the lower the equilibrium replacement rate is. In particular, with λ arbitrarily small, the equilibrium approaches the highest replacement rate compatible with some agents being employed, ρ^{max}.

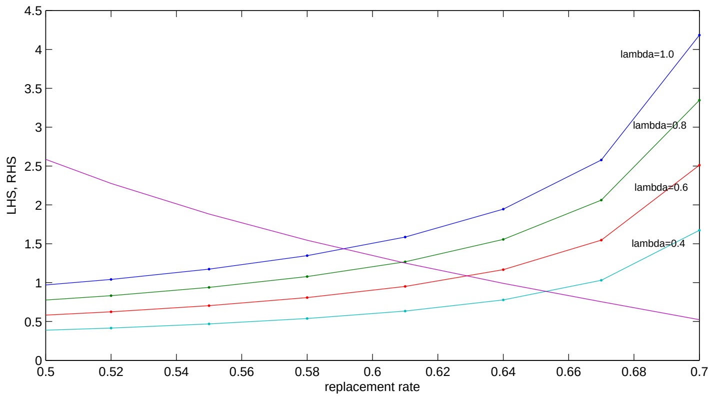

Figure 2: Impact of λ on the equilibrium replacement rate: a numerical illustration.

2.8 Welfare analysis

We here explore the welfare implications of the equilibrium. Two economies identical in every respect, apart from their values of λ, will be characterized by different equilibrium replacement rates, unemployment rates, and levels of welfare. To rank these equilibria in terms of welfare, we simply compute the utilitarian criterion at each equilibrium. Let us define $W_{stat}(\rho)$ as the welfare in the absence of public choice, when ρ is indefinitely held constant:

$$W_{stat}(\rho) = \Upsilon(\rho)V_u(\rho, \tau(\rho)) + (1 - \Upsilon(\rho))\, V_e(\rho, \tau(\rho)) - g(\pi(\rho))$$

where $\Upsilon(\rho)$ is the unemployment rate when ρ remains constant. As there is no unemployment persistence in the baseline economy, we have:

$$\Upsilon(\rho) = \hat{u}_{\lambda=1}(\rho)$$

Indeed, in the baseline case, next period's unemployment rate only depends on the agents expectations regarding next period's replacement rate. When $\lambda = 1$, agents know with certainty next period's replacement rate ρ, and their current search effort is identical to that in the stationary equilibrium where the replacement rate would remain constant. It is then straightforward to notice that maximizing $W_{stat}(\rho)$ is equivalent to imposing the equilibrium condition in the no surprise model, where the public choice applies only from $t = 1$ on[9].

Let us denote as ρ^* the replacement rate maximizing $W_{stat}(\rho)$. When the condition in proposition 2.4 is met, the no surprise case equilibrium exists and is unique. Moreover, there is a single value of ρ solution of equation (7) and the second order condition is verified, which is equivalent to the function $W_{stat}(\rho)$ having a unique local and global maximum. In the baseline case, we have shown that the equilibrium replacement rate will be higher than in the no surprise case for all $\lambda > 0$, and that it is decreasing with λ. Consequently, the shorter is the expected duration of the deviation, the smaller is the welfare in the baseline case. Even if the deviation is expected to last forever, the baseline equilibrium will yield a welfare lower than that corresponding to ρ^*, as long as $\beta < 1$.

3 Time consistent choice of unemployment insurance

In this section, we build a model which is calibrated, and numerically simulated, in order to take explicitly the time-consistency dimension into account. The previous section has underlined a channel through which the frequency of the public choice affects the equilibrium outcome. Analytical results were obtained under fairly usual assumptions, but this approach, for tractability purposes (i) was deprived of any form of unemployment persistence, (ii) lacked the mechanisms which could explain how the initial equilibrium is reached and (iii) was deprived of repeated choices. The model developped here, while remaining as close as possible to the analytical one, deals with these three issues, thus giving rise to an additional channel through which the frequency of the public choice affects the equilibrium.

[9]In the no surprise case, the public choice leaves the $t = 0$ instantaneous utilities unchanged. If we assume that $\lambda = 1$, -but in sub-section 2.7, we have proved that the equilibrium replacement rate in the no surprise model is unique and independent from λ- maximizing the utilitarian criterion at $t = 0$ in the no surprise case is equivalent to maximizing $\beta \times W_{stat}(\rho)$.

3.1 The model

The economy consists of a continuum of households, who are either employed, or unemployed. Households preferences are represented by a time-separable additive expected utility, which writes:

$$E_0 \left(\sum_{t=0}^{\infty} u(c_t) - \zeta(s_t) \right)$$

where $u(c_t)$ is the instantaneous utility at date t, s_t is the search effort at date t and $\zeta(s_t)$ is the utility cost of search. Only agents currently unemployed provide the search effort, which affects the probability of being hired at the beginning of the following period, $\pi(s_t)$. Employed agents face an exogenous job destruction rate δ at the beginning of each period.

When employed, agents earn the after-tax wage $w(1 - \tau_t)$ where τ_t is the time t tax rate and w is the exogenous wage. When unemployed, agents receive net benefits $\rho_t w(1 - \tau_t)$. This definition, which contrasts with that of the previous section, makes the interpretation of the replacement rate easier, and it avoids the definition of an upper bound ρ^{max} on the replacement rate. Savings are precluded, and agents consume their entire income at each date.

The budget of the public insurance agency is balanced at every date. The tax rate solves:

$$\rho_t w U_t = \tau_t \left(w(1 - U_t) + \rho_t w U_t \right) \Leftrightarrow \tau_t = \frac{\rho_t U_t}{1 - U_t(1 - \rho_t)}$$

We are here constructing a recursive equilibrium such that the choice on the replacement rate, repeatedly operated by the insurance agency, satisfies the principle of time consistency. When currently choosing the replacement rate, the insurance agency knows that, in the future, choices shall be decided again, and that it cannot commit to future choices. Rather, it will take future choices, conditional on the future state of the economy, as given, and act accordingly.

In order to handle time-consistency, the insurance agency and all agents need to know the choice rule, $\Phi(U)$, which associates, to each possible state of the economy (exhaustively described here by the unemployment rate), the chosen replacement rate, and the law of motion of the economy, represented by the function $\Gamma(U, \rho')$. The structure of the policy choice is the following:

- at each date t, there is a probability $1 - \lambda$ that a new choice shall be currently decided upon. When decided, the next choice applies to all agents, not only the newly unemployed,
- a new choice, operated at date t, applies from date $t + T_a$ on.

Our baseline case corresponds to $T_a = 0$: the replacement rate shock, whenever chosen, applies immediately. However, to neutralize the pure redistributive effect described in the previous section, we also consider the case for $T_a = 1$: in this case, a policy shock decided at the beginning of date t, applies from $t + 1$ on. Given the time structure of the policy choice in the baseline case, the agent's program writes:

$$\begin{aligned}
V\left(e,U_t,\rho\right) &= u(w\left(1-\tau(U,\rho)\right)) + \beta\left[(1-\delta)\left(\lambda V\left(e,U',\rho\right)\right.\right. \\
&\quad + (1-\lambda)\ V\left(e,U',\Phi(U')\right)) + \delta\left(\lambda V\left(u,U',\rho\right)\right. \\
&\quad \left.\left. + (1-\lambda)\,V\left(u,U',\Phi(U')\right)\right)\right] \\
\text{s.t.} & \\
&U' = \Gamma\left(U,\rho\right) \\
V\left(u,U,\rho\right) &= \max_s\ \ u(\rho_t w\left(1-\tau(U,\rho)\right)) - \zeta(s) \\
&\quad +\beta\left[\pi(s)\left(\lambda V\left(e,U',\rho\right) + (1-\lambda)\,V\left(e,U',\Phi(U')\right)\right)\right. \\
&\quad \left. +(1-\pi(s))\left(\lambda\,V\left(u,U',\rho\right) + (1-\lambda)\,V\left(u,U',\Phi(U')\right)\right)\right] \\
\text{s.t.} & \\
&U' = \Gamma\left(U,\rho\right)
\end{aligned} \tag{9}$$

With probability $1-\lambda$ at each period, the insurance agency chooses a new replacement rate, until the next choice. It maximizes the utilitarian criterion evaluated at date t:

$$\Phi(U) = \arg\max_{\tilde{\rho}} \left\{UV\left(u,U,\tilde{\rho}\right) + (1-U)V\left(e,U,\tilde{\rho}\right)\right\}$$

The equilibrium consists of the choice rule $\Phi\left(U\right)$ and the law of motion for unemployment $\Gamma\left(U,\rho'\right)$ such that:

1. Given the law of motion for unemployment and the choice rule , $V\left(\varepsilon,U,\rho\right), \varepsilon = e,u$ is the value function solution to program (9), and $s\left(u,U,\rho\right)$ is the effort rule,

2. Given the effort rule, and for any state of the economy (U), and for any current replacement rate ρ, next period's unemployment rate, U', implied by the effort rule, is consistent with the expected law of motion $\Gamma\left(U,\rho\right)$,

3. Given the above value function, the maximization of the utilitarian criterion at each date is consistent with the expected policy rule $\Phi\left(U\right)$:

$$\forall U, \Phi\left(U\right) = \arg\max_{\tilde{\rho}} \left(1-U\right)V\left(e,U,\tilde{\rho}\right) + UV\left(u,U,\tilde{\rho}\right),$$

Note that in the case of the policy announcement ($T_a = 1$), the model structure somewhat differs from the above presentation (see Appendix D for a complete presentation of this version of the model).

3.2 Calibration

As this model is numerically computed[10] (see Appendix E for a presentation of the numerical algorithm), all functions need be specified, and all parameters need be assigned

[10]Numerically, we resort to the standard method of grid discretization. We define the interval consisting of all possible values for the unemployment rate $[U_{min}; U_{max}]$ and the interval for the replacement rate $[\rho_{min}; \rho_{max}]$ where $U_{min}, U_{max}, \rho_{min}, \rho_{max}$ are chosen so as to ensure that the equilibrium values for the unemployment rate and the replacement rate belong to these two intervals.

values. The two terms of the instantaneous utility function are assumed to be:

$$u(c) = \frac{C^{1-\gamma}}{1-\gamma}, \zeta(s) = As^{\psi}$$

where γ is the relative risk aversion. The exit rate function writes (a common formulation,as, for example, in Wang and Williamson (1996)):

$$\pi(s) = 1 - e^{-\xi s}$$

We use a model periodicity of three weeks to capture the average unemployment duration on the labor market[11]. The model is calibrated on the US segment of low-qualified workers. We focus on this segment for two main reasons. First, low-qualified workers earn rather frequently the minimum wage, which is consistent with our assumption regarding the exogeneity of the gross wage. Secondly, this segment of the population has a strong need for unemployment benefit as a means to smooth consumption, as opposed to other households who save on other grounds -mainly life cycle motives- and who could smooth consumption more easily on their own. Precisely, we have to set values for $\gamma, \beta, \xi, \psi, \delta$ and w. Without any loss of generality, the wage rate w can be normalized to 1. The relative risk aversion γ is set to a value of 2[12].

The value of the discount factor β is somewhat more problematic in this simple setting. Indeed, among the remaining parameters to be calibrated, ξ, ψ and δ, directly affecting labor market flows, will be calibrated in order to match some key labor market statistics (see below). Only β remains. As its interactions with labor market flows are far from obvious, it would seem awkward to try to calibrate it, by imposing another labor market statistic to match. From our point of view, this parameter would be best calibrated, jointly with the interest rate, in a model where agents can save, and in order to match financial assets statistics. As, here, savings are precluded, we are not given this possibility. Consequently, we have decided to rely on a different version of this model, allowing for savings, and developed in a companion paper (Kankanamge and Weitzenblum (2014)). In this other model with savings, the interest rate is set according to the portfolio composition of low-qualified workers (based on the Survey of Consumer Finances (SCF), 2007), and the discount factor is calibrated in order to match the average asset holdings (also obtained from the SCF 2007) of low-qualified workers. We find $\beta = 0.9945$. For lack of any other solution, and since the two models are structurally identical[13], we have decided to impose here also the value $\beta = 0.9945$.

ξ, δ and ψ directly affect the search intensity of the unemployed agents. Their calibration is therefore based on what we regard as the key quantitative properties of the U.S. labor market for low-qualified workers. Precisely, we intend to reproduce (i) the unemployment rate for this type of workers, (ii) the average unemployment duration and (iii) the elasticity of the average unemployment duration with respect to the replacement

[11]The model period needs be considerably smaller thant the average duration of unemployment, which, in the case of the US, is close to one quarter.

[12]This falls in the range of values usually admitted for this parameter, due to the wide range of estimates in the literature.

[13]Apart from the saving behavior which is absent here and present in the companion paper. The values of the different parameters are of course different, but the specifications and the imposed labor market statistics are all identical.

Parameter	Value	Description
w	1	Wage (normalization)
β	0.9945	Discount factor
γ	2	Relative risk aversion
δ	0.0133	Job destruction rate
ξ	0.2149	Exit rate parameter
ψ	1.9391	Curvature of effort function

Table 1: Benchmark calibration values

rate. We use the Current Population Survey (CPS)[14] to compute values for the first two elements. We retain CPS data from mid-2006 to mid-2007. We find an unemployment rate of about 7% for this fringe of the population. The unemployment duration is about 17 weeks in the data. Given the unemployment rate and the unemployment duration, it is straightforward to deduce the average employment duration, which in turn implies a job destruction rate δ of 0.0133. Finally, the estimation of the elasticity of unemployment duration with respect to the replacement rate does not lead to a consensus. A positive value below 1 is to be expected (see Layard et al. (1991)). For low-qualified workers in the U.S., we assume that the elasticity should be on the lower side of the plausible values (calculated for various countries) and set to a target value of 0.4. We find that with $\psi = 1.9391$ and $\xi = 0.2149$, we match the above mentioned calibration targets.

This calibration has been implemented in a simpler model deprived of the public choice: an exogenous replacement rate is assumed to be held indefinitely, and the economy has converged toward its long-run equilibrium. The baseline economy corresponds to a replacement rate equal to 40%. The elasticity of unemployment duration is computed by simulating the counterfactual experiment of marginally increasing the replacement rate, which yields its associated unemployment duration, and comparing it to its baseline counterpart.

3.3 Simulation results

3.3.1 Equilibrium replacement rates

Table 2 presents, for various expected choice periodicities (the periodicity being the inverse of $1 - \lambda$), the associated equilibrium replacement rate and unemployment rate. The lines denoted as $T_a = 1$ correspond to the policy announcement: with an identical calibration, we simulate the model, following the equilibrium definition of Appendix D.

Choice periodicity (years)		$\lambda = 0$	0.25	0.5	1	4	10
	Replacement rate (in %)	–	82.5	78.1	75.2	72.5	71.9
$T_a = 0$	Unempl. rate (in %)	–	10.8	10.2	9.8	9.5	9.4
	Replacement rate (in %)	88.4	80.2	76.9	74.6	72.3	71.8
$T_a = 1$	Unempl. rate (in %)	12.1	10.5	10.0	9.7	9.5	9.4

Table 2: Time-consistent unemployment benefits

[14]Data were taken from http://www.nber.org/cps/

The equilibrium replacement rate, in the case of the policy announcement ($T_a = 1$), depends on the frequency of the public choice, and is decreasing with the choice periodicity. The analytical model developed in the previous section has shown that the gains from immediate redistribution make the equilibrium replacement rate lower, the longer the periodicity. As the pure redistributive channel is completely offset here ($T_a = 1$), this result proves that there exists another mechanism explaining why the equilibrium replacement rate depends on the periodicity of the public choice. Precisely, the analytical model, for the purpose of simplicity, was deprived of any form of unemployment persistence: tomorrow's unemployment rate depended on the expected replacement rate, and was independent from today's unemployment. This property justified why the redistributive channel was the only mechanism operative in the model. When unemployment persistence is accounted for, another channel, which will be thoroughly analyzed and commented below, appears. Table 2 shows that in the baseline case, the two effects, which operate in the same direction, tend to make the replacement rate even more periodicity dependent.

For a choice deterministically repeated every period ($\lambda = 0$), the replacement rate is not defined in the baseline case, while it remains well below 100% ($\rho = 88.4\%$) when $T_a = 1$. Indeed, in the baseline case, as the analytical model has shown, the temptation to benefit from the current redistribution is too strong, and no replacement rate strictly below 100% could be an equilibrium. This is no more the case, when the redistributive channel is neutralized.

Quantitatively, the periodicity has a considerable impact on the level of the replacement rate: that it may vary of some $10 - 15$ percentage points, on pure time-consistency issues, is indeed highly significant. It needs however be noted that the replacement rate depends on the periodicity of the public choice mostly for very low periodicities (3 months, 6 months and one year). Such low periodicities may not seem very realistic, when it comes to thinking of re-designing -and, therefore, bringing recurrently into the political debate- a structural policy such as unemployment insurance. If one were to regard the periodicity of nation-wide elections as the natural one for the choice of unemployment insurance, then 4 years could be regarded as a central value. Bringing down the periodicity to 1 year would consequently generate a replacement rate increase of around two percentage points. Finally, we also note that the equilibrium replacement rate, for all periodicities, remains well above the one observed in the U.S. economy. In this stylized setup where agents do not have access to financial assets, and each household is assumed to earn a single wage, the impact of labor income risk and the opportunity to insure agents better are magnified.

3.3.2 Model dynamics

To improve our understanding of the dynamic forces at work here, we provide the coefficients of a linear approximation of the two core functions (the law of motion of unemployment, and the public choice rule) around the equilibrium.[15] We define the following

[15] Analyzing the impact of the public choice periodicity on the shape of the functions is considerably easier with this quantitative characterization, than with graphic representations.

slopes [16]:

$$\alpha_1^U = \left.\frac{\Delta U_{t+1}}{\Delta U_t}\right|_{\rho=\rho^{equ}} \qquad \alpha_2^U = \left.\frac{\Delta U_{t+1}}{\Delta \rho_t}\right|_{U=U^{equ}}$$

$$\alpha_1^\rho = \frac{\Delta \rho_t}{\Delta U_t}$$

The table 3 below presents these 3 coefficients for various public choice periodicities.

Choice periodicity (years)	0.25	1	4
α_1^U	0.8748	0.8628	0.8588
α_2^U	0.0062	0.0112	0.0136
α_1^ρ	-0.1048	-0.0470	-0.0158

Table 3: Law of motion and choice rule for various choice periodicities

Let us first analyze the values for a given periodicity, say 4 years. The dynamics of the unemployment rate are characterized by considerable persistence, as could be expected for such a stock variable: α_1^U is close to 1. The coefficient α_2^U is positive: an increase in the replacement rate, decided at the current period, reduces the incentive to search for a job. Regarding the choice rule, the coefficient α_1^ρ is negative. The impact of the current unemployment level on the public choice depends on (i) the insurance and redistributive effects and (ii) how the disincentive effect is affected by the unemployment rate. For higher unemployment levels, the insurance/redistributive effect of a marginal increase in benefits should mechanically be higher. Considering only the pure redistribution operated at the current period, it is rather intuitive to notice that the utilitarian gains from a marginal increase in the replacement rate will be higher, if the proportion of unemployed benefiting from it is larger. The disincentive effect is quantitatively assessed in the simulations as follows: the partial derivative α_2^U can be numerically computed for various unemployment rates, and not only at the equilibrium. It appears that α_2^U is higher when the unemployment rate is higher[17]: therefore, the disincentive effect is larger, for higher unemployment rates. These two channels tend to have opposite effects on the opportunity, in the utilitarian sense, to raise or lower benefits. Here, the impact of unemployment on the disincentive effect dominates, but as we will see below, the magnitude of the impact of α_1^ρ is very small.

Turning to the impact of the periodicity on the various coefficients, the longer the periodicity, the higher the impact of the replacement rate on the future unemployment rate (the coefficient α_2^U). This is an illustration of how the disincentive effect at a given date is all the higher, as the expected duration of the benefit increase is longer. It is a pure dynamic effect, quantifying the unemployment rate change between two consecutive dates. If the periodicity of the public choice increases, not only will a replacement rate

[16]The functions being computed on the grid, we simply choose two gridpoints around the equilibrium. For the law of motion, we choose two gridpoints on either side of the equlibrium with respect to one dimension -U or ρ-, keeping the other dimension constant, and draw the line. For the public choice rule, we simply choose two gridpoints around the equilibrium unemployment rate, and draw the line.

[17]The values are not reproduced in this article.

increase last on average longer, but the unemployment rate will react more, from one time period to the next. Although the two models are somewhat different, this is the quantitative counterpart of the $\hat{u}'(\rho')$ function of the model developed in section 2. In both cases, as the periodicity increases, so does the marginal impact of a given replacement rate increase on the unemployment rate change. The effect of the periodicity on the coefficient α_1^ρ is less straightforward. It implies that, for a given unemployment rate increase, the chosen replacement rate will be lower, as the periodicity gets shorter.

To provide another illustration of the dynamics of the public choice, figure 3 plots 4 time paths, corresponding to the dynamics of the replacement rate, in case a marginal increase of 1.5 percentage points is exogenously operated at date $t = 0$, starting from the time-consistent equilibrium, and for a public choice periodicity of 6 months. By construction, the time-consistent equilibrium is such that, once it has been reached, no further deviation will be regarded as beneficial, in the utilitarian sense. However, for such an opinion to be grounded, the public insurance agency -and all agents in this economy- need to assess the impact of a marginal deviation on the dynamics of this economy, to make sure that such a deviation is not worth it. The dynamics simulated here precisely consists of the path of the economy, which is initially located at the time-consistent equilibrium, which is exogenously hit by a small shock on the replacement rate, and which converges back to its equilibrium. Figure 3 presents some time paths, among the great diversity of which agents would expect, in case an upward deviation of the replacement rate was implemented.

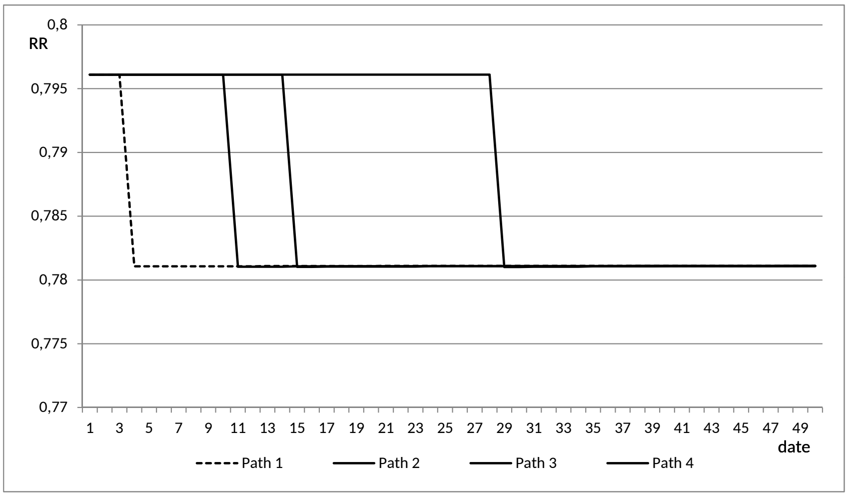

Figure 3: Dynamics of the replacement rate

The initial level of the replacement rate consists of the equilibrium value (78.1%, for a 6 months periodicity) to which an exogenous 1.5 points increase is added. As the public choice occurs at random, these 4 different paths correspond to different realizations of the random variable representing the occurrence of a new public choice at each date. Once a public choice first occurs, the new replacement rate will be reset at a value very close to its initial level. This means that the choice on ρ does not depend much on what makes the state of the economy vary with time, that is, on the unemployment rate.

Although this is not perceptible on the graph, the first endogenous public choice slightly overshoots the final equilibrium replacement rate. This reflects the fact that the public choice rule is a decreasing function of the unemployment rate ($\alpha_1^\rho < 0$). Following the initial exogenous replacement rate increase, the unemployment rate rises. As previously mentioned, when the unemployment rate is higher, so is the disincentive effect. Consequently, when the first endogenous public choice occurs, the public authority is faced with a higher disincentive effect, so it decides to be a little less generous. Quantitatively, this channel is very weak -the overshooting is not visible on the graph-, and in particular, it has no repercussion on the stability of the equilibrium.

The fact that the endogenous choice of the public insurance agency is extremely close to its equilibrium value suggests that the equilibrium definition of the analytical model, in sub-section 2.4, however naive, should lead to quantitative results, very close to a repeated public choice approach[18]. Indeed, agents were naive in the theoretical approach, because they expected the replacement rate to be forever reset at a constant value, once the initial deviation would no longer apply. Here, agents expect the public insurance agency future choices correctly, but quantitatively, these choices hardly differ from the initial replacement rate.

3.3.3 Temptation to deviate and duration of the deviation: another channel

We here bring to light the channel which explains why the equilibrium replacement rate in the baseline model is periodicity dependent. As the structure of the baseline model is rather complex, it is fruitful to consider simpler dynamics, deprived of the recursive structure necessary to handle time consistency. The effects on unemployment and welfare of various deviations differing in their duration, can then be more easily analyzed. Precisely, we shall consider here deterministic temporary deviations in the replacement rate, perfectly foreseen by all agents. The transitional dynamics are characterized as follows:

- the initial state of the economy consists of the unique unemployment rate U_0 consistent with the replacement rate ρ being indefinitely imposed in the past,
- at date $t = 0$, a deviation of the replacement rate of magnitude $\Delta\rho$, effective from date $t = 1$ on (but known from the beginning of date $t = 0$), and lasting S periods, is implemented. This deviation is perfectly anticipated by all agents,
- from date $t = S + 1$ on, the replacement rate reverts back to ρ, its initial value.

The deviation, decided upon at date $t = 0$, starts from date $t = 1$ on. Therefore, the redistributive channel, underlined in the previous section, is completely offset. In this time-varying environment, the recursive formulation of the agents' program is:

$$\begin{aligned} V(e,t) &= u(w(1-\tau_t)) - \zeta(0) + \beta\left[(1-\delta)V(e,t+1) + \delta V(u,t+1)\right] \\ V(u,t) &= \max_{s_t} u(\rho_t w(1-\tau_t)) - \zeta(s_t) + \beta\left[\pi(s_t)V(e,t+1) + (1-\pi(s_t))V(u,t+1)\right] \end{aligned} \tag{10}$$

The dynamics of the economy then consists of $\left((\rho_t)_{t\geqslant 0}, (\tau_t)_{t\geqslant 0}, (s_t)_{t\geqslant 0}, (U_t)_{t\geqslant 0}\right)$ such that:

[18]This does not mean that the quantitative results in this section should be close to the numerical illustration of subsection 2.6, because the model structures are not identical.

- given the paths $(\rho_t)_{t\geqslant 0}$ and $(\tau_t)_{t\geqslant 0}$, $(s_t)_{t\geqslant 0}$ is the optimal search effort of the program (10),
- the path for unemployment, $(U_t)_{t\geqslant 0}$ is the only one compatible with U_0 and search effort $(s_t)_{t\geqslant 0}$. That is:

$$\forall t \geqslant 0, U_{t+1} = \delta(1 - U_t) + (1 - \pi(s_t))U_t$$

- given $(\rho_t)_{t\geqslant 0}, (\tau_t)_{t\geqslant 0}$ and $(U_t)_{t\geqslant 0}$, the budget of the insurance agency is balanced at every date:

$$\forall t \geqslant 0, \tau_t = \frac{\rho_t U_t}{1 - U_t(1 - \rho_t)}$$

To highlight in what way the gains and costs arising from a marginal increase of the replacement rate depend on the duration of the deviation, we proceed as follows. First, we search for the initial replacement rate ρ such that a deviation of magnitude $\Delta\rho = 2$ percentage points, effective for a single model period (date $t = 1$), leaves unchanged the utilitarian criterion evaluated at $t = 0$. In other words, ρ is the numerical counterpart of the equilibrium replacement rate defined in sub-section 2.4, and for a deterministic deviation lasting one period[19]. We then simulate the transition, starting with the same initial unemployment rate U_0, for various durations S. In particular, we focus on the differentiate impact of deviations lasting 1 and 2 periods.

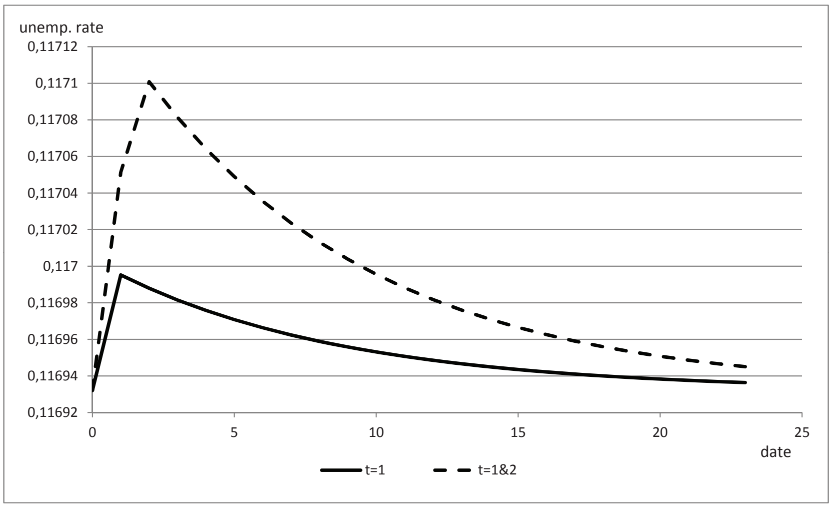

Figure 4: Unemployment rate dynamics

From figure 4, it is clear that the cumulated impact on the unemployment rate of a deviation lasting 2 periods is considerably higher than twice the impact of a deviation lasting one period. Indeed, the initial unemployment rate increase is almost twice as high

[19]Indeed, in this setting, the public choice is decided only at $t = 0$, never repeated thereafter, and the initial state of the economy is such that the deviation is not beneficial according to the utiliarian criterion.

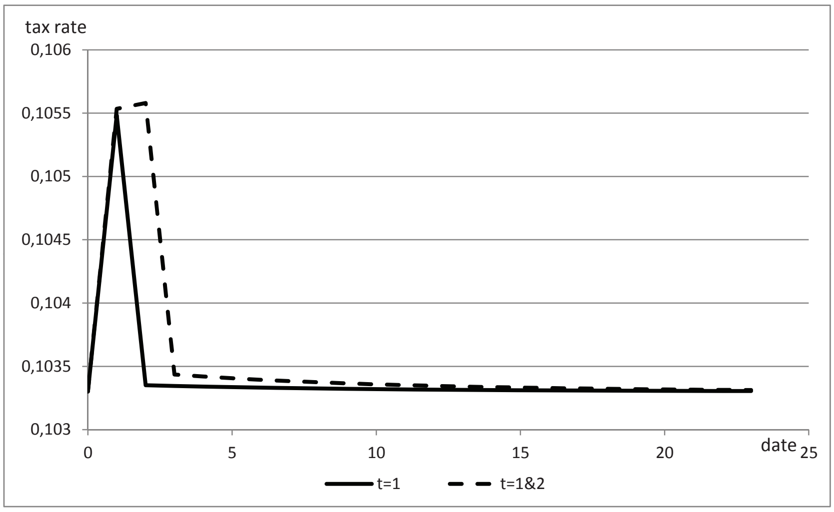

Figure 5: Tax rate dynamics

when $S = 2$, and it keeps increasing at date $t = 2$, whereas it has already started to decrease from date $t = 1$, when the deviation ends at $t = 1$.

Let us focus on the case where $S = 2$. The increase in the replacement rate lowers the intertemporal utility gain from finding a job, $V(e, t+1) - V(u, t+1)$. For marginal variations, the unemployment increase is proportional to the search effort reduction, which itself is proportional to the decrease in this utility gain. Ignoring for the purpose of the argument the impact of the unemployment increase on the tax rate and on the after-tax income, the difference $V(e, 1) - V(u, 1)$ diminishes, both because the difference in instantaneous utility $u(w(1-\tau_1) - u(\rho_1 w(1-\tau_1))$ falls, and because the future expected difference $u(w(1-\tau_2) - u(\rho_2 w(1-\tau_2))$ falls too. This latter term is discounted by $\beta(1 - \delta - \pi(s_2))$ [20].

Consequently, the search effort reduction at date $t = 0$ is driven by the replacement rate increases at both dates. At date $t = 1$, only the replacement rate increase effective at the next date affects the search effort. This justifies why the unemployment rate increase from $t = 0$ to $t = 1$ is roughly twice as large as that from $t = 1$ to $t = 2$.

As seen from date $t = 0$, the increase in ρ at $t = 2$ doubly affects the unemployment rate (by lowering the search effort at $t = 0$ and $t = 1$), while the $t = 1$ increase affects it only once, through its impact on date $t = 0$ search effort. Figure 4 shows that the unemployment rate increase at date $t = 2$ (and at all subsequent dates) is around three times larger when the shock on the replacement rate lasts two periods, instead of one. Therefore, the marginal disincentive costs increase more than proportionately with the duration.

As for the marginal insurance gains, holding the unemployment rate constant, and ignoring the discount rate, they are proportional to the duration of the policy shock. As long as the shock still applies, these gains arise from the gap in marginal utilities between employed and unemployed agents, the gap remaining constant.

Therefore, the marginal disincentive costs increase more than proportionately with the

[20] to account for the fact that, as seen from $t = 1$, the gain is all the smaller, as the agent may find a job at $t = 2$ (with probability $\pi(s_2)$)

duration, while the marginal insurance gains are approximately proportional to the duration. As the duration increases, marginal costs increase faster than gains. In the example presented in the figures 4 and 5, the initial state corresponds to a maximal utilitarian criterion, for a deviation lasting 1 period. This means that the marginal gains and costs exactly offset each other. If the deviation lasted 2 periods, the marginal costs would become larger than the marginal gains. The net impact on the utilitarian criterion would be negative, and, on the opposite, a replacement rate reduction would be beneficial. The current replacement rate would then be too high, if the deviation were to last 2 periods, instead of one.

Figure 5, although less striking at first sight, consists in another illustration of the mechanism, through its impact on the tax rate. When the deviation lasts for 2 periods, the tax rate is already higher at date 1. It is not very clear from the graph, because the largest part of the tax rate increase is due to the increase in the replacement rate, the magnitude of which is identical in both cases. The unemployment rate increase, which, within one period, remains very modest, only marginally contributes to the tax rate increase. At date $t = 2$, the tax keeps on increasing. From date $t = 2$ on, the tax rate is distinctly higher than for a deviation lasting one period.

Therefore, the forward-looking nature of workers behavior explains why the impact of an announced replacement rate increase is greater, when the increase will take place in a more distant future. This, in turn, justifies why the periodicity of the public choice in the replacement rate negatively affects the equilibrium replacement rate, even in the absence of the redistributive channel described in section 2.

3.4 Time consistent choice of unemployment benefits time profile

Apart from the redistributive channel, we have shown that the generosity of the unemployment insurance depends on the duration of the public choice because the disincentive impact of benefits is all the stronger, as it is expected to apply later. One could argue that this mechanism does not pertain directly and only to the duration of the public choice, but also to the time profile of unemployment benefits. In other words, for an agent newly unemployed at date t, what makes a marginal increase in benefits at date $t + 2$ more costly in terms of disincentive, than a marginal increase at date $t + 1$, is not only the fact that the deviation lasts for 2 time periods, but also that it applies for agents who have been unemployed for 2 periods. This suggests that the previous result -the fact that the equilibrium replacement rate is lower, when the public choice is less frequent- would deserve to be reformulated in terms of equilibrium time profile. It could be that the frequency of the public choice affects the degressivity of the time profile, rather than the average level of benefits.

To answer this question in quantitative terms, we let the insurance agency impose replacement rates which depend on the duration of the current unemployment spell. We therefore distinguish short-term unemployed agents from long-term ones. We assume that agents always start a new unemployment spell as short-term unemployed. Then, at the beginning of each new period, they become long-term unemployed with probability η. This implies that the length of time spent in the short-term state is stochastic, and its mean, conditional on not finding a job, is equal to $\frac{1}{\eta}$. We therefore need to take into account two levels of replacement rate (ρ_s and ρ_l respectively denoting the short-term and the long term replacement rates) and two pools of unemployed: the short term unemployment U_s and the long-term one, U_l. Apart from this distinction, the model structure (preferences, exit rate function) remains unchanged. The budget of the insurance system is balanced at every date t, so that:

$$\rho_{s,t} w U_{s,t} + \rho_{l,t} w U_{l,t} = \tau_t \left(w(1 - U_{s,t} - U_{l,t}) + \rho_{s,t} w U_{s,t} + \rho_{l,t} w U_{l,t} \right)$$
$$\Leftrightarrow \tau_t = \frac{\rho_{s,t} U_{s,t} + \rho_{l,t} U_{l,t}}{1 - U_{s,t}(1-\rho_{s,t}) - U_{l,t}(1-\rho_{l,t})}$$

The agent's program is (ignoring the time subscript t):

$$\begin{aligned}
V(e, U_s, U_l, \rho_s, \rho_l) \quad & = u(w\,(1 - \tau(U_s, U_l, \rho_s, \rho_l))) + \beta\,[\\
& (1-\delta)\left(\lambda V\left(e, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\ V\left(e, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right) \\
& +\delta\left(\lambda V\left(s, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(s, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right)] \\
V(s, U_s, U_l, \rho_s, \rho_l) \quad & = \max_s \ u(\rho_s w\,(1 - \tau(U_s, U_l, \rho_s, \rho_l))) - \zeta(s) \\
& +\beta\left[(1-\eta)\left(\pi(s)\left(\lambda V\left(e, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(e, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right)\right.\right. \\
& +(1-\pi(s))\left(\lambda\ V\left(s, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(s, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right)) \\
& +\eta\left(\pi(s)\left(\lambda V\left(e, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(e, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right)\right. \\
& \left.\left. +(1-\pi(s))\ \left(\lambda\ V\left(l, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(l, U'_s, U'_l, \Phi_s(U'_s, U'l), \Phi_l(U'_s, U'l)\right)\right)\right)\right] \\
V(l, U_s, U_l, \rho_s, \rho_l) \quad & = \max_s \ u(\rho_l w\,(1 - \tau(U_s, U_l, \rho_s, \rho_l))) - \zeta(s) \\
& +\beta\left[\pi(s)\left(\lambda V\left(e, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(e, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right)\right. \\
& \left. +(1-\pi(s))\left(\lambda V\left(l, U'_s, U'_l, \rho_s, \rho_l\right) + (1-\lambda)\,V\left(l, U'_s, U'_l, \Phi_s(U'_s, U'_l), \Phi_l(U'_s, U'_l)\right)\right)\right] \\
\text{s.t.} \quad & \\
& U'_s = \Gamma_s\left(U_s, U_l, \rho_s, \rho_l\right) \\
& U'_l = \Gamma_l\left(U_s, U_l, \rho_s, \rho_l\right)
\end{aligned} \tag{11}$$

In line with the above models, the public insurance agency operates a new choice at every period, with probability $1-\lambda$. It is assumed that the company simultaneously chooses the two replacement rates in order to maximize the utilitarian criterion, as shown here:

$$\begin{aligned}
(\Phi_s(U_s, U_l), \Phi_l(U_s, U_l)) = \arg\max_{\tilde{\rho}_s, \tilde{\rho}_l} \{ & U_s V\left(s, U_s, U_l, \tilde{\rho}_s, \tilde{\rho}_l\right) + U_l V\left(l, U_s, U_l, \tilde{\rho}_s, \tilde{\rho}_l\right) \\
& +(1 - U_s - U_l) V\left(e, U_s, U_l, \tilde{\rho}_s, \tilde{\rho}_l\right)\}
\end{aligned}$$

The equilibrium definition (see appendix F) very closely parallels that of section 3.1. Simply, the law of motion of the economy (resp. the choice rule) now consists of two functions, Γ_s and Γ_l (resp. Φ_s and Φ_l) instead of one in case of a flat replacement rate.

In order to make the different models as comparable as possible, we keep the calibration parameters of sub-section 3.2. We mainly focus on the impact of the choice periodicity on the time profile and the average level of the replacement rate, but it is also useful to analyze the impact of η, the probability of moving from short term unemployed to long term unemployed. Of course, this parameter should, in principle, be treated like the replacement rate, as it is another feature of the unemployment insurance. However, to avoid facing too large a choice -which would be both technically challenging and difficult to analyze-, we choose to regard η as a constant parameter. Simulating the public choice for different ηs can be considered as a robustness test of our model, which requires an exogenous η.

Table 4 below presents the optimal time-consistent replacement rates for short and long term unemployed for $\eta = 0.25$ (implying an average duration in short-term unemployment of one quarter) and various values for λ.

Choice periodicity (years)	0.25	0.5	1	4
ρ_s(in %)	91.4	91.5	91.9	94.2
ρ_l(in %)	79.5	74.4	71.0	70.8
Unempl. rate (in %)	10.4	9.8	9.5	9.4
Average replacement rate	83.6	80.6	78.8	79.6

Table 4: Time-consistent unemployment benefits time profiles

The degressivity is significantly affected by the periodicity of the public choice. As could be expected, the long term replacement rate falls as the periodicity increases. This confirms the intuition, which emerged at sub-section 3.3.3, that with a higher λ, the probability that the current increase in the replacement rate will still apply in a few model periods is higher, generating a larger impact on the unemployment rate dynamics, and consequently, bringing down the optimal benefit level. What was not expected is the non negligible impact of λ on the short term unemployment benefit: it increases as the choice becomes less frequent, thereby exacerbating the impact of the public choice periodicity on the degressivity of the time profile. The fact that the degressivity is much smaller, when choices occur more frequently, is not very surprising. Indeed, the degressivity pattern is optimal because, as seen from today, an increase in benefits in the near future (say, 4 periods) has a larger disincentive effect than an increase in the immediate future (say, 1 period), provided that the increase is expected to last long enough to be still effective in the near future. If not, agents simply expect the replacement rate to be chosen again before reaching this term. In this case, the long term unemployed replacement rate does not exert such a large disincentive effect, and there is less need for the optimal time profile to be degressive.

The impact of λ on the average replacement rate is much less pronounced than on the time profile, as the opposite evolutions of the two replacement rates tend to offset each other. It does not evolve monotonically with the choice periodicity. The fact that the choice periodicity affects the degressivity of the benefit time-profile much more than the average replacement rate echoes the well known result, that the optimal design of unemployment insurance involves subtle time-profile considerations more than the definition of a global level of generosity. What is original here is that the magnitude of the degressivity is significantly affected by time-consistency issues.

Simulations[21] with different values for η show that the impact of the periodicity of the public choice on the degressivity of the time-profile remains qualitatively identical, and quantitatively very close to the one presented here.

4 Concluding remarks

In this article, we have brought to light mechanisms, which help understand in what way the time-horizon characterizing the choice of unemployment insurance affects the temptation of the social planner to marginally deviate toward higher levels of benefits and, in turn, the equilibrium replacement rate. Balancing short-run pure redistributive gains with median run efficiency costs is a possible channel. The forward-looking nature of the search behavior of unemployed is at the core of another channel. When considering flat profiles for the replacement rate, a marginal increase of the replacement rate will trigger efficiency costs and insurance gains, which are not equally affected by the time horizon of the public choice. Efficiency costs (or, equivalently, disincentive effects) grow faster than insurance gains with the considered periodicity, so that a social planner with a longer horizon will be less tempted to increase unemployment benefits, and will end up with a lower equilibrium replacement rate. When allowing for more flexible time-profiles for the replacement rate, it appears that the equilibrium time-profile is always degressive, whatever the periodicity of the choice, but the degressivity is positively affected by the choice periodicity.

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[21]The results are not reproduced in the article, but are available upon request.

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A The stationary equilibrium model

In this appendix, we sketch the model of an economy similar to that of section 2 where the replacement rate (denoted as ρ' here) remains constant and is not subject to temporary deviations, and we present some properties of its equilibrium. In this case, recalling that the expression of τ as a function of ρ' and U is[22].

$$\tau(\rho', U) = \tau(\rho', 1-\pi) = \frac{\rho'(1-\pi)}{\pi} = \frac{\rho'}{\pi} - \rho'$$

the optimality condition on the search effort writes:

$$g'(\pi) = \beta\left(u(w(1 - \frac{\rho'}{\pi} + \rho')) - u(\rho' w)\right) \tag{12}$$

This expression is defined as long as (i) the tax rate remains below 100% and (ii) the net income of the unemployed is smaller than the net income of the employed. It is easy to check that the second requirement is tighter than the first, and can be rewritten as:

$$\pi \geqslant \rho'$$

The left-hand side of the optimality condition is an increasing function, with $g'(0) = 0$. It is also convex, as $g'''(\pi) > 0$. One can check that the right-hand side is a strictly concave function of π. The right-hand side is only defined for $\pi \geqslant \rho'$ and for $\pi = \rho'$, it is equal to zero. Therefore, the two sides either (i) cross twice, (ii) tangentially admit a common value or (iii) never cross[23]. Figure 6 represent the most frequent case where the two curves cross twice.

As ρ' increases, the right-hand side moves toward the right, and there exists an upper bound on ρ' where the two curves tangentially admit a single common point. For higher ρ', no equilibrium exists.

Out of the two intersections, only that corresponding to the higher value for π consists in a stable equilibrium. To understand it more easily, we adopt a different graphical representation of the optimality condition:

$$\pi_1 = g'^{-1}\left(\beta\left(u(w(1 - \frac{\rho'}{\Pi_1} + \rho')) - u(\rho' w)\right)\right) \Leftrightarrow \pi_1 = \gamma(\Pi_1)$$

where π_1 is the effort of a given agent and Π_1 is the effort of all the other agents. It defines an increasing and concave function of Π_1. At the equilibrium, $\pi_1 = \Pi_1$. Graphically, either the curve and the $\pi_1 = \Pi_1$ line cross twice, or they tangentially meet once, or they do not have any common value. Out of the two intersections, the first one corresponds to an unstable equilibrium. As soon as the effort Π_1 deviates from the equilibrium value, each individual effort will diverge by a greater amount. On the opposite, the second

[22]In this model, all time subscripts will be dropped as we are analyzing a stationary equilibrium where all aggregate variables remain constant throughout the time.

[23]We know that $g'(0) = 0$, that $g''(0) < +\infty$ and from equation (12), we can see that the derivative of the right-hand side can be made arbitrarily high at $\pi = \rho'$, by choosing ρ' small enough. Therefore, there is always an equilibrium, provided that the replacement rate is small enough. For a given replacement rate, it could however be that the two curves do not cross.

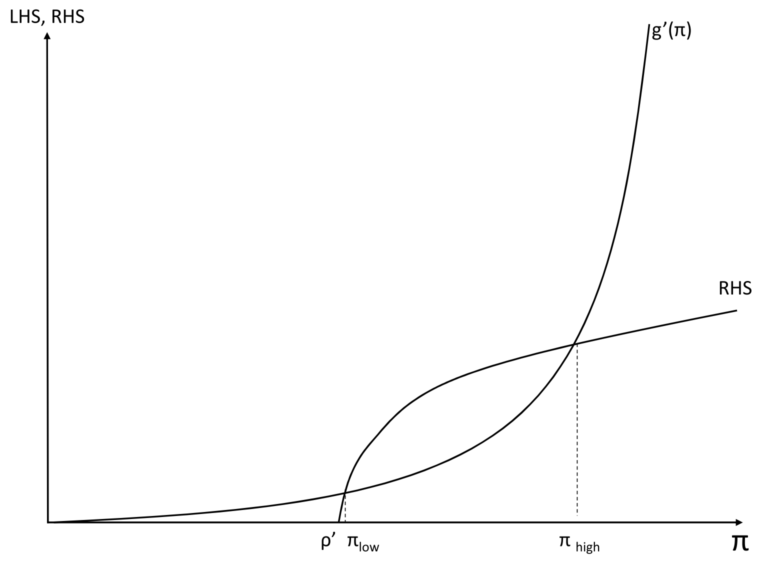

Figure 6: Optimality condition representation in the stationary model: the frequent case

intersection is stable in the sense that if Π_1 slightly deviates from its equilibrium value, all individual efforts will converge back to the equilibrium[24]. In the case where the curve tangentially meets the $\pi_1 = \Pi_1$ line, the equilibrium is not stable, as any downward deviation will make the sequential process diverge.

At the stable equilibrium, the slope of the curve is strictly smaller than one, which is equivalent to:

$$g''(\pi) > \frac{\beta\rho' w}{\pi^2} u'(w(1 - \frac{\rho'}{\pi} + \rho'))$$

Note that the the model developed in section 2 endogenizes the replacement rate, but given the replacement rate, all other properties of the equilibrium developed in this appendix, in terms of comparative statics, will be verified. Therefore, the above equation ensures that the denominator in the right-hand side of the expression (3) is strictly positive.

Also, note that as ρ' increases, at any given level of effort π,the $\gamma(\pi)$ curve is moving downward, ensuring that the stationary effort π is, lower, that is, the stationary unemployment rate is higher. Finally, as the function $\gamma(\pi)$ is trivially continuous wherever it is defined, the function $\Upsilon(\rho')$ is continuous.

[24]This would become clearer if we defined a sequential equilibrium where each individual effort was a function of the past aggregate effort: $\pi_1^{t+1} = \gamma(\Pi_1^t)$ and if we described its associated dynamics.

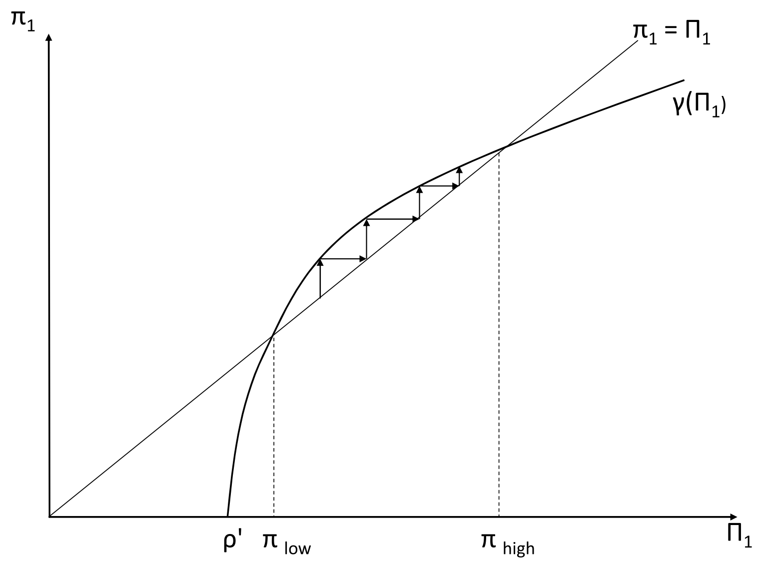

Figure 7: Optimality condition in the stationary model: alternative representation

B Proofs

B.1 Proof of Proposition 2.1

Let us start with the optimality condition (2). We have:

$$\frac{\partial V_e(\rho', U_0)}{\partial \rho'} = V_{e1}(\rho', U_0) = -\tau_1'(\rho', U_0)wu'(w(1-\tau(\rho', U_0))$$
$$+\beta\lambda\pi_1 V_{e1}(\rho', \hat{u}(\rho')) + \beta\lambda\pi_1\hat{u}'(\rho')V_{e2}(\rho', \hat{u}(\rho'))$$
$$+\beta(1-\lambda)\pi_1\hat{u}'(\rho')V_{e2}(\overline{\rho}, \hat{u}(\rho')) + \beta\lambda(1-\pi_1)V_{u1}(\rho', \hat{u}(\rho')).$$

where the number in the subscript refers to the position of the variable which is derived. This results follows from $\frac{\partial V_u(\rho', U)}{\partial U} = V_{u2}(\rho', U) = 0$ (variations in the unemployment rate only affect the after tax wage, and not the replacement income (nor does it affect the search effort). The envelop condition writes:

$$\frac{dV_i(\rho', U_0)}{d\rho'} = \frac{\partial V_i(\rho', U_0)}{\partial \rho'} + \frac{\partial \pi_1}{\partial \rho'}\underbrace{\frac{\partial V_i(\rho', U_0)}{\partial \pi_1}}_{=0} \quad for\ i = e, u$$

We also have:

$$\frac{\partial V_e(\rho', U)}{\partial U} = V_{e2}(\rho', U) = -w\tau_2'(\rho', U)u'(w(1-\tau(\rho', U)))$$

Finally, we have:

$$V_{u1} = wu'(\rho' w) + w\tau_1'(\rho', \hat{u}(\rho'))u'(w(1-\tau(\rho', \hat{u}(\rho')))) + V_{e1}(\rho', \hat{u}(\rho'))$$

We define $\Phi(\rho', \hat{u}(\rho')) = wu'(\rho' w) + w\tau_1'(\rho', \hat{u}(\rho'))u'(w(1-\tau(\rho', \hat{u}(\rho'))))$. The utilitarian criterion writes:

$$\frac{\partial W_0(\rho', U_0)}{\partial \rho'} = (1-U_0)V_{e1}(\rho', U_0) + U_0 V_{u1}(\rho', U_0) = V_{e1}(\rho', U_0) + U_0\Phi(\rho', U_0)$$

The first order condition can be further simplified by using the conditions: $\rho' = \overline{\rho}$ and $\hat{u}(\rho') = U_0$ which hold at the equilibrium when the insurance agency decides not to deviate. It follows that:

$$\begin{aligned} V_{e1}(\rho', U_0) = -\tau_1'(\rho', U_0)wu'(w(1-\tau(\rho', U_0)) + \beta\lambda V_{e1}(\rho', U_0) + \beta(1-\pi_1)\lambda\Phi(\rho', U_0) \\ +\beta\pi_1\hat{u}'(\rho')V_{e2}(\rho', U_0) \end{aligned}$$

$$\begin{aligned} \Rightarrow (1-\beta\lambda)V_{e1}(\rho', U_0) = -\tau_1'(\rho', U_0)wu'(w(1-\tau(\rho', U_0)) \\ +\beta(1-U_0)\hat{u}'(\rho')(-w\tau_2'(\rho', U_0))u'(w(1-\tau(\rho', U_0))) + \beta U_0\lambda\Phi(\rho', U_0) \\ \Rightarrow (1-\beta\lambda)\left[V_{e1}(\rho', U_0) + U_0\Phi(\rho', U_0)\right] = -\tau_1'(\rho', U_0)wu'(w(1-\tau(\rho', U_0)) \\ +\beta(1-U_0)\hat{u}'(\rho')(-w\tau_2'(\rho', U_0))u'(w(1-\tau(\rho', U_0))) + U_0\Phi(\rho', U_0) \end{aligned}$$

Therefore, when the FOC is attained, one has:

$$\begin{aligned} -\tau_1'(\rho', U_0)wu'(w(1-\tau(\rho', U_0)) + \beta(1-U_0)\hat{u}'(\rho')(-w\tau_2'(\rho', U_0))u'(w(1-\tau(\rho', U_0))) \\ +U_0\Phi(\rho', U_0) = 0 \\ \Leftrightarrow -\frac{U_0}{1-U_0}wu'(w(1-\tau(\rho', U_0))) - \beta(1-U_0)\hat{u}'(\rho')w\frac{\rho'}{(1-U_0)^2}u'(w(1-\tau(\rho', U_0))) \\ +U_0\left(wu'(\rho' w) + \frac{U_0}{1-U_0}wu'(w(1-\tau(\rho', U_0)))\right) = 0 \end{aligned}$$

which is equivalent to the condition expressed in the proposition.

B.2 Proof of Proposition 2.2

From equation (4), one gets:

$$\begin{aligned} g''(\pi_1)\frac{d\pi_1}{d\rho'} = \beta\lambda\left[-wu'(\rho' w) - w\tau_1'(\rho', \hat{u}(\rho'))u'(w(1-\tau(\rho', \hat{u}(\rho'))))\right. \\ \left.+\hat{u}'(\rho')(V_{e2}(\rho', \hat{u}(\rho')) - V_{u2}(\rho', \hat{u}(\rho')))\right] + \beta(1-\lambda)\hat{u}'(\rho')\left(V_{e2}(\overline{\rho}, \hat{u}(\rho')) - V_{u2}(\overline{\rho}, \hat{u}(\rho'))\right) \end{aligned}$$

As all agents provide the same search effort, one trivially has: $\pi_1 = 1-\hat{u}(\rho')$ which implies that:$\frac{d\pi_1}{d\rho'} = -\hat{u}'(\rho')$. Given that $V_{u2}(\rho', \hat{u}(\rho')) = V_{u2}(\overline{\rho}, \hat{u}(\rho')) = 0$, given the expression for V_{e1}, one obtains:

$$\begin{aligned} -g''(1-\hat{u}(\rho'))\hat{u}'(\rho') = \beta\lambda\left[-wu'(\rho' w) - w\frac{\hat{u}(\rho')}{1-\hat{u}(\rho')}u'(w(1-\tau(\rho', \hat{u}(\rho'))))\right. \\ \left.-\hat{u}'(\rho')\frac{w\rho'}{(1-\hat{u}(\rho'))^2}u'(w(1-\tau(\rho', \hat{u}(\rho'))))\right] \\ +\beta(1-\lambda)\left(-\hat{u}'(\rho')\frac{w\overline{\rho}}{(1-\hat{u}(\rho'))^2}u'(w(1-\tau(\overline{\rho}, \hat{u}(\rho'))))\right) \end{aligned}$$

At the equilibrium, $\rho' = \overline{\rho}$. Regrouping the two terms containing the disincentive effect $\hat{u}'(\rho')$ in the left-hand side, the expression stated in the proposition obtains.

The denominator is necessarily strictly positive: $g''(1-U_0) - \frac{\beta\rho' w}{(1-U_0)^2} u'(w(1-\tau(\rho', U_0))) > 0$. As shown in the appendix A, it follows from the fact that it is equal to $g''(1-U_0)\left(1 - \frac{d\pi_1}{d\Pi_1}\right)$ where π_1 is the effort of a given household, and Π_1 is the average effort of all households. Economically, a household's effort depends, among others, on other households' effort. A necessary condition for obtaining a stable equilibrium is that the derivative $\frac{d\pi_1}{d\Pi_1}$ be smaller than 1.

B.3 Proof of Proposition 2.3

Assume here that $\lim_{c \to 0} u'(c) = +\infty$. For $\rho' \underset{\rho'>0}{\longrightarrow} 0$, it is clear that the tax rate becomes arbitrarily small, the marginal utility of consuming the net wage is bounded below, while the marginal utility of consuming the unemployment compensation becomes arbitrarily large. Hence, the left-hand side tends to infinity.

As for the right-hand side, given the assumption $g'''(\pi) > 0$, as $\rho' \longrightarrow 0$, $U_0 \searrow$, and $g''(1-U_0) \nearrow$. One also checks that $\frac{\beta\rho' w}{(1-U_0)^2} u'(w(1-\tau(\rho', U_0))) \searrow$. Therefore, the denominator is strictly bounded below. As for the numerator, it is the sum of 2 terms: that with the marginal utility of employed workers is bounded above as $\rho' \longrightarrow 0$. The other one, with $u'(\rho' w)$, is the product of a term bounded above and $\rho' u'(\rho' w)$. As $\rho' \longrightarrow 0$, this term will necessarily become arbitrarily small, as compared to $\frac{u'(\rho' w)}{u'(w(1-\tau(\rho', U_0)))}$. If the marginal utility of consumption remains bounded at $c = 0$, then the left-hand side is continuous at $\rho' = 0$, and is strictly positive, while the right-hand side tends to 0.

For $\rho' \longrightarrow \rho_{max}$, the left-hand side, being simply a continuous function, tends toward its value for $\rho' = \rho_{max}$. As for the right-hand side, the numerator also tends to a finite value, while the denominator tends to 0 (this corresponds to the highest possible replacement rate of the equilibrium developed in Appendix A, where the two curves meet tangentially only once). Consequently, the right-hand side becomes arbitrarily large.

B.4 Proof of Proposition 2.4

Let us rewrite equation (6) by multiplying it by $\frac{u'(w(1-\tau(\rho', U_0)))}{u'(\rho' w)}$. We obtain:

$$\frac{u'(\rho' w) - u'\left(w(1-\tau(\rho', U_0))\right)}{u'(\rho' w)} = \frac{\rho' \beta^2 \lambda w \left(\frac{u'(w(1-\tau(\rho', U_0)))}{1-U_0} + \frac{U_0 [u'(w(1-\tau(\rho', U_0)))]^2}{u'(\rho' w)(1-U_0)^2}\right)}{U_0 \left(g''(1-U_0) - \frac{\beta\rho' w}{(1-U_0)^2} u'(w(1-\tau(\rho', U_0)))\right)} \tag{13}$$

Regarding the right-hand side, consider first the denominator of the fraction. The assumption ensures that, as $\pi = 1 - U_0$ increases, $(1-\pi)g''(\pi)$ increases too. This implies that $U_0 g''(1-U_0)$ is decreasing in U_0. The second term, $U_0 \frac{\beta\rho' w}{(1-U_0)^2} u'(w(1-\tau(\rho', U_0)))$, increases as U_0 increases, thus garanteeing that the denominator decreases with U_0. We know, from appendix A, that the denominator is always positive, and tends to 0 as ρ' approaches ρ^{max}. As for the numerator, one can easily check that it is strictly increasing in ρ'. Therefore, the right-hand side is an increasing function of ρ'.

As for the left-hand side of (13), it is a strictly decreasing function of ρ': as ρ' increases, so does the tax rate, the after-tax wage is reduced, and the marginal utility of employed workers rises. Conversely, the marginal utility of unemployed workers declines.

If the two sides of (13) should intersect, they would do it at most once. From proposition 2.3, we already know that the equation (6) will hold at least for one value of ρ', which is equivalent to stating that equation (13) will hold at least once, too. Therefore, the equilibrium exists, and is unique. With the assumption $(1-\pi)g'''(\pi) > g''(\pi)$, we know that there is a unique solution, so that, in the graph 1, we know that, for all $0 < \lambda \leqslant 1$, there is a unique intersection between the two curves. Note, however, that the above proof does not imply that the right-hand side of equation (6) is strictly increasing.

C Announced policy shocks in the analytical model

We here present the model when a policy shock decided at $t = 0$, applies only from $t = 1$ on. The recursive formulation of the value functions here requires to add a third variable: ρ, the predetermined replacement rate, which holds for sure at the date considered, needs be distinguished from ρ', which will hold for sure at the next date. At date $t = 0$, the replacement rate remains $\overline{\rho}$. It is equal to ρ' for sure at date $t = 1$. From $t = 1$ on, if the current replacement rate is still ρ', it will remain constant at the next date with probability λ. Otherwise, it will revert back to $\overline{\rho}$ for ever. In any case, the value of the future replacement rate is known one period ahead.

The search effort, as in the model developed in section 2, only depends on next period's replacement rate ρ'.[25] The value functions of the agents are:

$$
\begin{aligned}
V_e(\rho,\rho',U) = \max_{\pi_1}\{&u(w(1-\tau(\rho,U))) - g(\pi_1) + \beta[\pi_1(\lambda V_e(\rho',\rho',\hat{u}(\rho')) + (1-\lambda)V_e(\rho',\overline{\rho},\hat{u}(\rho'))) \\
&+ (1-\pi_1)(\lambda V_u(\rho',\rho',\hat{u}(\rho')) + (1-\lambda)V_u(\rho',\overline{\rho},\hat{u}(\rho')))]\} \\
V_u(\rho,\rho',U) = \max_{\pi_1}\{&u(\rho w) - g(\pi_1) + \beta[\pi_1(\lambda V_e(\rho',\rho',\hat{u}(\rho')) + (1-\lambda)V_e(\rho',\overline{\rho},\hat{u}(\rho'))) \\
&+ (1-\pi_1)(\lambda V_u(\rho',\rho',\hat{u}(\rho')) + (1-\lambda)V_u(\rho',\overline{\rho},\hat{u}(\rho')))]\}
\end{aligned}
$$

We here need to compute 3 partial derivatives for each employment status. The following ones can be immediately computed:

$$
\begin{aligned}
\frac{\partial V_e(\rho,\rho',U)}{\partial \rho} &= V_{e1}(\rho,\rho',U) = -w\tau_1'(\rho,U)u'(w(1-\tau(\rho,U))) \\
\frac{\partial V_e(\rho,\rho',U)}{\partial U} &= V_{e3}(\rho,\rho',U) = -w\tau_2'(\rho,U)u'(w(1-\tau(\rho,U))) \\
\frac{\partial V_u(\rho,\rho',U)}{\partial \rho} &= V_{u1}(\rho,\rho',U) = wu'(\rho w) \\
\frac{\partial V_u(\rho,\rho',U)}{\partial U} &= V_{u3}(\rho,\rho',U) = 0
\end{aligned}
$$

[25] This follows from the separability of the instantaneous utility function in consumption and search effort: the effort is independent from the current consumption level.

Regarding the remaining one, we note that:

$$\frac{\partial V_e\left(\rho, \rho', U\right)}{\partial \rho'} = V_{e2}\left(\rho, \rho', U\right) = V_{u2}\left(\rho, \rho', U\right)$$

We further characterize this derivative, evaluated at date $t = 0$:

$$\begin{aligned}
V_{e2}\left(\overline{\rho}, \rho', U_0\right) = \beta \left[\lambda \left(\pi_1 \hat{u}'(\rho') V_{e3}(\rho', \rho', \hat{u}(\rho'))\right.\right. \\
+\pi_1 V_{e1}(\rho', \rho', \hat{u}(\rho')) + \pi_1 V_{e2}(\rho', \rho', \hat{u}(\rho')) \\
+(1-\pi_1)\hat{u}'(\rho') V_{u3}(\rho', \rho', \hat{u}(\rho')) + (1-\pi_1) V_{u1}(\rho', \rho', \hat{u}(\rho')) + (1-\pi_1) V_{u2}(\rho', \rho', \hat{u}(\rho'))) \\
+(1-\lambda)\left(\pi_1 \hat{u}'(\rho') V_{e3}(\rho', \overline{\rho}, \hat{u}(\rho')) + (1-\pi_1)\hat{u}'(\rho') V_{u3}(\rho', \overline{\rho}, \hat{u}(\rho'))\right. \\
\left.\left.+\pi_1 V_{e1}(\rho', \overline{\rho}, \hat{u}(\rho')) + (1-\pi_1) V_{u1}(\rho', \overline{\rho}, \hat{u}(\rho'))\right)\right]
\end{aligned}$$

The equilibrium, as defined in 2.4, imposes that $\rho' = \overline{\rho}$ and $\hat{u}(\rho') = U_0$. The above expression then simplifies as:

$$\begin{aligned}
(1-\beta\lambda)V_{e2} = \beta\lambda\pi_1 \hat{u}'(\rho')\left(-w\tau_2'(\rho', U_0) u'(w(1-\tau(\rho', U_0)))\right) \\
+\beta\pi_1\left(-w\tau_1'(\rho', U_0) u'(w(1-\tau(\rho', U_0)))\right) \\
+\beta(1-\pi_1) w u'(\rho' w) + \beta(1-\lambda)\pi_1 \hat{u}'(\rho')\left(-w\tau_2'(\rho', U_0) u'(w(1-\tau(\rho', U_0)))\right)
\end{aligned}$$

Using the expressions for τ_1' and τ_2', and the fact that $\pi_1 = 1 - U_0$, we obtain:

$$\begin{aligned}
(1-\beta\lambda)V_{e2}(\overline{\rho}, \rho', U_0) = -\frac{\beta w \rho'}{1-U_0}\hat{u}'(\rho') u'(w(1-\tau(\rho', U_0))) \\
+\beta w U_0\left(u'(\rho' w) - u'(w(1-\tau(\rho', U_0)))\right)
\end{aligned}$$

This leads to this version of the Baily-Chetty condition:

$$\frac{u'(\rho' w) - u'\left(w(1-\tau(\rho', U_0))\right)}{u'\left(w(1-\tau(\rho', U_0))\right)} = \frac{\rho' \hat{u}'(\rho')}{U_0(1-U_0)}$$

As for $\hat{u}'(\rho')$, it is easy to see that the first order optimality condition corresponds to that of the baseline model, with $\lambda = 1$.

D The case of announced policy shocks with repeated choices in the simulated model

We here present the time consistent model in the case where a policy shock decided at t only applies from $t+1$ on. Apart from this, all other assumptions regarding agents' preferences, job flows, and the repeated public choice, remain unchanged.

We however need to keep track of two different replacement rates: ρ, which is currently applying at date t, and ρ', known as of date t, but applying from $t+1$ on. It is easy to verify that the search effort is independent from the current replacement rate, and depends on all future replacement rates (among which, only tomorrow's is known with certainty). This guarantees that the law of motion of the economy depends on next period's replacement rate only. The modified agent's program writes:

$$
\begin{aligned}
V\left(e,U_t,\rho,\rho'\right) \quad &= u(w\left(1-\tau(U,\rho)\right)) + \beta\left[(1-\delta)\left(\lambda V\left(e,U',\rho',\rho'\right)\right.\right. \\
&\quad + (1-\lambda)\ V\left(e,U',\rho',\Phi(U')\right)) + \delta\left(\lambda V\left(u,U',\rho',\rho'\right)\right. \\
&\quad \left.\left. + (1-\lambda)\, V\left(u,U',\rho',\Phi(U')\right)\right)\right] \\
\text{s.t.} \quad & \\
&U' = \Gamma\left(U,\rho'\right) \\
V\left(u,U,\rho,\rho'\right) \quad &= \max_s \ u(\rho w\left(1-\tau(U,\rho)\right)) - \zeta(s) \\
&\quad + \beta\left[\pi(s)\left(\lambda V\left(e,U',\rho',\rho'\right) + (1-\lambda)\, V\left(e,U',\rho',\Phi(U')\right)\right)\right. \\
&\quad \left. + (1-\pi(s))\left(\lambda\, V\left(u,U',\rho',\rho'\right) + (1-\lambda)\, V\left(u,U',\rho',\Phi(U')\right)\right)\right] \\
\text{s.t.} \quad & \\
&U' = \Gamma\left(U,\rho'\right)
\end{aligned}
\tag{14}
$$

The modified utilitarian criterion evaluated at date t is:

$$
\Phi(U) = \arg\max_{\tilde{\rho}} \left\{UV\left(u,U,\rho,\tilde{\rho}\right) + (1-U)V\left(e,U,\rho,\tilde{\rho}\right)\right\}
$$

In particular, the chosen replacement rate is independent from the current one (ρ), because, as the insurance agency maximizes the welfare, the current replacement rate is predetermined; it only affects the current instantaneous utility of the agents with no interaction whatsoever with either the current search effort of the unemployed, or the expected part of the intertemporal utility.

The modified equilibrium consists of the choice rule $\Phi\left(U\right)$ and the law of motion for unemployment $\Gamma\left(U,\rho'\right)$ such that:

1. Given the law of motion for unemployment and the choice rule , $V\left(\varepsilon,U,\rho,\rho'\right), \varepsilon = e,u$ is the value function solution to program (14), and $s\left(u,U,\rho'\right)$ is the effort rule,

2. Given the effort rule, and for any state of the economy (U), and for any future replacement rate ρ', next period's unemployment rate, U', implied by the effort rule, is consistent with the expected law of motion $\Gamma\left(U,\rho'\right)$,

3. Given the above value function, the maximization of the utilitarian criterion at each date is consistent with the expected policy rule $\Phi\left(U\right)$:

$$
\forall U, \Phi\left(U\right) = \arg\max_{\tilde{\rho}} \left(1-U\right) V\left(e,U,\rho,\tilde{\rho}\right) + UV\left(u,U,\rho,\tilde{\rho}\right),
$$

E Numerical algorithm of the time-consistent equilibrium model

In this model, there are 3 state variables: the current unemployment rate U_t, the current replacement rate ρ_t and the current employment status of the considered agent, i. Apart from the employment status, which can take only 2 values (employed or unemployed), the other two variables are continuous ones. We resort to standard grid discretization techniques to approximate these variables. Uniform grids have been chosen: N_U (resp. N_ρ) points for the grid on U_t (resp. on ρ_t) as the range of possible values is not too wide. Moreover, once the equilibrium has been found, we can zoom in on these ranges, thus increasing the accuracy of our algorithm.

The algorithm consists of a fixed point in the following 2 rules: (i) the choice rule which relates the current public choice in terms of the replacement rate ρ_t to the current unemployment rate U_t, and (ii) the law of motion for the unemployment rate U_{t+1} which relates this variable (future value) to U_t (current value) and ρ_t.

As the aggregate state is exhaustively described by U_t and ρ_t, there are exactly $N_U \times N_\rho$ (resp. N_U) points where the law of motion for unemployment (resp. the public choice rule) has to be computed.

Our strategy consists in:

1. making an initial guess on values for the 2 rules over the grid,

2. given these rules, computing the value functions of the agents and the search effort of unemployed households,

3. given the search effort function, computing on the $N_U \times N_\rho$ grid the law of motion of unemployment and given the value functions, computing the public choice rule on the N_U grid by maximizing the utilitarian criterion,

4. comparing the obtained law of motion and public choice rule with the ones postulated *ex ante*. If they are close enough, stop, otherwise, update the *ex ante* functions by applying a relaxation method.

5. once the convergence is over, simulating the temporal path of the economy by starting with a given aggregate state (U_0, ρ_0), and iterating forward. At every period with probability $1 - \lambda$, let the public authority choose the replacement rate. Stop when the state of the economy has fully stabilized.

The 2 rules are not simultaneously updated. Rather, we proceed with nested loops, each of which being devoted to the convergence of one of the above 2 rules. Besides, in practice, the computed law of motion and public choice rule need be smoothed a little when they are updated. Indeed, the algorithm rests on iterating on and updating these functions a large number of times, and at each stage, the numerical computation is only valid up to a certain degree of accuracy. In the absence of this smoothing, a slight perturbation could grow bigger at each iteration and prevent the algorithm from converging properly.

The public choice rule is smoothed by calculating the OLS line and taking a weighted average of the obtained line and the calculated rule (relaxation method). The law of motion, for each level of the replacement rate, is smoothed by applying the OLS on the first difference of the function, and taking a weighted average of the obtained function and the computed one. Given that we are searching for a stable equilibrium, once it has been numerically found, we can zoom in on the two grid windows, so that the two functions can be approximated with a high precision by the linear and quadratic smoothing functions.

F Equilibrium definition of the time-consistent choice on the UI time-profile

The equilibrium consists of the choice rules $(\Phi_s(U_s, U_l), \Phi_l(U_s, U_l))$ and the laws of motion for unemployment $(\Gamma_s(U_s, U_l, \rho_s, \rho_l), \Gamma_l(U_s, U_l, \rho_s, \rho_l))$ such that:

1. Given the laws of motion for unemployment and the choice rules , $V(\varepsilon, U_s, U_l, \rho_s, \rho_l), \varepsilon = \{e, s, l\}$ is the value function solution to program (11), and $s(\varepsilon, U_s, U_l, \rho_s, \rho_l), \varepsilon = \{s, l\}$ is the effort rule,

2. Given the effort rule, and for any state of the economy (U_s, U_l), and for any current replacement rate time-profile (ρ_s, ρ_l), next period's unemployment rates, (U'_s, U'_l), implied by the effort rule, are consistent with the expected laws of motion $(\Gamma_s(U_s, U_l, \rho_s, \rho_l), \Gamma_l(U_s, U_l, \rho_s, \rho_l))$,

3. Given the above value function, the maximization of the utilitarian criterion at each date is consistent with the expected policy rule $(\Phi_s(U_s, U_l), \Phi_l(U_s, U_l))$:

$$(\Phi_s(U_s, U_l), \Phi_l(U_s, U_l)) = \arg\max_{\tilde{\rho}_s, \tilde{\rho}_l} \{U_s V(s, U_s, U_l, \tilde{\rho}_s, \tilde{\rho}_l) + U_l V(l, U_s, U_l, \tilde{\rho}_s, \tilde{\rho}_l)$$
$$+ (1 - U_s - U_l) V(e, U_s, U_l, \tilde{\rho}_s, \tilde{\rho}_l)\}$$